Question	Scheme	Marks	AOs
13	$\int \frac{x}{(2x+1)^3}\,\mathrm{d}x = \frac{x(2x+1)^{-2}}{-4} + \int \frac{(2x+1)^{-2}}{4}(\mathrm{d}x)$	M1	3.1a
	$= \ldots + \frac{(2x+1)^{-1}}{-8}$	dM1	1.1b
	$-\frac{x}{4(2x+1)^2} - \frac{1}{8(2x+1)}$	A1	1.1b
	e.g. $= \left(-\frac{2}{4(2\times 2+1)^2} - \frac{1}{8(2\times 2+1)}\right) - \left(-\frac{0}{4(2\times 0+1)^2} - \frac{1}{8(2\times 0+1)}\right)$	ddM1	1.1b
	e.g. $-\frac{2}{100} - \frac{1}{40} + \frac{1}{8} = \frac{2}{25}$ *	A1*	2.1
		(5)	
13 Alt I	$\int \frac{x}{(2x+1)^3}\,\mathrm{d}x = \int \frac{u-1}{4u^3}(\mathrm{d}u)$ where $u = 2x+1$	M1	3.1a
	$= \int \frac{1}{4}u^{-2} - \frac{1}{4}u^{-3}\,\mathrm{d}u = \ldots$	dM1	1.1b
	$= -\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}$	A1	1.1b
	$\int_0^2 \frac{x}{(2x+1)^3}\,\mathrm{d}x = \left[-\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}\right]_1^5 = \left(-\frac{1}{20} + \frac{1}{200}\right) - \left(-\frac{1}{4} + \frac{1}{8}\right)$	ddM1	1.1b
	e.g. $-\frac{1}{20} + \frac{1}{200} + \frac{1}{8} = \frac{2}{25}$ *	A1*	2.1
		(5)	
			(5 marks)

Notes:

M1: Obtains $\alpha x(2x+1)^{-2} \pm \beta \int (2x+1)^{-2}\ (\mathrm{d}x)$ o.e. where $\alpha, \beta \neq 0$ but may be equal to each other (you do not need to be concerned about how they arrive at this)

dM1: Uses a correct method to integrate an expression of the form
$\pm\beta\int(2x+1)^{-2}\ (\mathrm{d}x) \to \pm\gamma(2x+1)^{-1}, \quad \beta, \gamma \neq 0$
It is dependent on the previous method mark.

A1: $-\frac{x}{4(2x+1)^2} - \frac{1}{8(2x+1)}$ o.e. Allow this to be unsimplified

Watch out for the DI method

	D	**I**
+	x	$(2x+1)^{-3}$
−	1	$-\frac{(2x+1)^{-2}}{4}$
+	0	$\frac{(2x+1)^{-1}}{8}$

Giving correct integration e.g. $\int \frac{x}{(2x+1)^3}\,\mathrm{d}x = -\frac{x(2x+1)^{-2}}{4} - \frac{(2x+1)^{-1}}{8}$

Score M1dM1 for obtaining $\pm px(2x+1)^{-2} \pm q(2x+1)^{-1},\ \ p,q \neq 0$ and A1 for both correct.

ddM1: Substitutes 0 and 2 into an expression of the form $\pm px(2x+1)^{-2} \pm q(2x+1)^{-1},\ \ p,q \neq 0$ or equivalent and subtracts either way round. Evidence of limits used cannot be the given answer. Condone slips but not directly evaluating the expression as 0 when x is 0. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the dx throughout

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Alternative method I – substitution using $u = 2x+1$

M1: Uses a suitable substitution e.g. $u = 2x+1$ and proceeds to $A\int \frac{u-1}{u^3}\,(\mathrm{d}u)$ o.e.

dM1: Splits into separate fractions and attempts to integrate $A\int u^{-2} - u^{-3}\,(\mathrm{d}u)$ Look for at least one correct index for one of the two terms i.e. $u^{-2} \to u^{-1}$ or $u^{-3} \to u^{-2}$. It is dependent on the previous method mark.

A1: $-\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}$ o.e.

ddM1: Substitutes correct limits (1 and 5 if in terms of u) into an expression of the correct form $...u^{-1} + ...u^{-2}$ (or may have substituted back in terms of x and substitutes in 0 and 2 into an expression of the correct form $\pm p(2x+1)^{-2} \pm q(2x+1)^{-1},\ \ p,q \neq 0$ o.e and subtracts either way round – see above for ddM1). Evidence of limits used cannot be the given answer. Condone slips. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the du throughout

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Alternative method II – partial fractions

M1: Writes $\frac{x}{(2x+1)^3}$ as $\frac{0.5}{(2x+1)^2} - \frac{0.5}{(2x+1)^3}$ o.e. Allow $\pm\frac{M}{(2x+1)^2} \pm \frac{N}{(2x+1)^3}$ (where M and N are constants)

dM1: $\int \frac{x}{(2x+1)^3}(\mathrm{d}x) = \int \frac{\text{"0.5"}}{(2x+1)^2}(\mathrm{d}x) - \int \frac{\text{"0.5"}}{(2x+1)^3}(\mathrm{d}x) = \pm...(2x+1)^{-1} \pm ...(2x+1)^{-2}$

A1: $-\frac{1}{4(2x+1)^1} + \frac{1}{8(2x+1)^2}$ o.e.

ddM1: Substitutes 0 and 2 into an expression of the correct form $\pm...(2x+1)^{-1} \pm ...(2x+1)^{-2}$ and subtracts either way round. Evidence of limits used cannot be the given answer. Condone slips but not directly evaluating the expression as 0 when x is 0. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the dx throughout.