Question	Scheme	Marks	AOs
15 (a)	$\frac{1}{2}r^2\theta+\frac{1}{10}r^2=240 \Rightarrow r\theta=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ or $\theta=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r^2}$	M1 A1	3.4 1.1b
	Substitutes into the expression for P $r\theta=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ into $(P=)\ r\theta+2r+\frac{1}{5}r$	dM1	3.4
	$P=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r}+2r+\frac{1}{5}r=\frac{480}{r}-\frac{1}{5}r+2r+\frac{1}{5}r=2r+\frac{480}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{\mathrm{d}P}{\mathrm{d}r}=\right)2-\frac{480}{r^2}$	M1	1.1b
	Sets $\frac{\mathrm{d}P}{\mathrm{d}r}=0\Rightarrow r^2=240$ $r=$ awrt 15.5	dM1 A1	2.1 1.1b
		(3)	
(c)	$\left(\frac{\mathrm{d}^2P}{\mathrm{d}r^2}=\right)\frac{960}{r^3}$	M1	1.1b
	$\left(\frac{\mathrm{d}^2P}{\mathrm{d}r^2}=\right)$ awrt $0.26>0$ proving a minimum value of P	A1	1.1b
		(2)	

(9 marks)

Notes:

(a) **Note that just finding a correct equation for the area and/or a correct equation for the perimeter (before any substitution) is insufficient to score any marks.**

M1: Uses area formulae to form an equation of the form $\alpha r^2\theta+\beta r^2=240$ o.e. $(\alpha,\beta\neq 0)$ and rearranges to make $r\theta$, θ or $r\theta+\frac{1}{5}r$ the subject. Look for:

$$r\theta=\frac{M\pm Nr^2}{r}\left(=\frac{M}{r}\pm Nr\right)\text{ o.e. or }\theta=\frac{M\pm Nr^2}{r^2}\left(=\frac{M}{r^2}\pm N\right)\text{ o.e. where } M,N\neq 0$$

or $r\theta+\frac{1}{5}r=\frac{L}{r}$ $L\neq 0$ o.e. May work in degrees.

A1: A correct rearrangement for θ or $r\theta$ or $r\theta+\frac{1}{5}r$ which may be unsimplified (may be in degrees)

$$r\theta=\frac{240-\frac{1}{10}r^2}{\frac{1}{2}r}\text{ o.e. e.g. } r\theta=\frac{2400-r^2}{5r}\text{ or } r\theta=\frac{480-0.2r^2}{r}$$

or $r\theta+\frac{1}{5}r=\frac{480}{r}$ o.e.

or $\theta = \dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r^2}$ o.e. e.g. $\theta = \dfrac{2400-r^2}{5r^2}$ or $\theta = \dfrac{480}{r^2} - \dfrac{1}{5}$ or $\theta = 480r^{-2} - 0.2$

dM1: Substitutes their $r\theta = \dfrac{M \pm Nr^2}{r}$ o.e. or $\theta = \dfrac{M \pm Nr^2}{r^2}$ o.e. or $r\theta + \dfrac{1}{5}r = \dfrac{L}{r}$ into an expression of the form $(P =)\ r\theta + Qr$, $Q \neq 0$ (typically $P = r\theta + \dfrac{11}{5}r$) which may be unsimplified or in degrees. It is dependent on the previous method mark. It is acceptable for their valid expression for θ, $r\theta$ or $r\theta + \dfrac{1}{5}r$ to be substituted into the perimeter expression directly (without first seeing them in the perimeter expression).

A1*: $P = 2r + \dfrac{480}{r}$ following a correct method (condone slips to be recovered) and all previous marks scored. Condone invisible brackets to be recovered.
$P =$, Perimeter $=$ must be seen at least once in their solution in the correct place.

(b) **Mark (b) and (c) together. There is no requirement to see the notation $\dfrac{dP}{dr}$ in part (b). It may even be called $\dfrac{dy}{dx}$. Allow use of e.g. P' or e.g. y'**

M1: $\left(\dfrac{dP}{dr} =\right) p \pm \dfrac{q}{r^2}$ where p and q are non-zero constants

dM1: Sets or implies that their $\dfrac{dP}{dr} = 0$ and proceeds to $mr^{\pm 2} = n, \quad m \times n > 0$. It is dependent on the previous method mark. Do not be concerned by the mechanics of the rearrangement. This mark may be implied by a correct answer to their $p - \dfrac{q}{r^2} = 0$. You may need to check this on your calculator.

A1: $r = \text{awrt } 15.5$ or $\sqrt{240}\ \left(= 4\sqrt{15}\right)$ Do not accept $\pm$ (ignore any units if given)

(c) **Condone other letters used instead of *P* and *r* for $\dfrac{d^2P}{dr^2}$ e.g. $\dfrac{d^2y}{dx^2}$ for M1 only.**

Just using $\dfrac{dP}{dr}$ and considering a sign change is M0A0

M1: Differentiates and finds $\left(\dfrac{d^2P}{dr^2} =\right) \pm \dfrac{f}{r^3}$ (do not be concerned about the sign)

A1: **Note if they score A0 in (b) then this mark cannot be scored.**
Requires

- a correct a correct expression for $\dfrac{d^2P}{dr^2}$
- a correct value for $\left(\dfrac{d^2P}{dr^2} =\right) \dfrac{960}{r^3} = \text{awrt } 0.26$ using awrt 15.5 (but allow 0.23(43..) if using 16)
- a correct comparison with 0 **and** a conclusion e.g. minimum

The expression for the second derivative does not need to be labelled but if it is then it must be $\dfrac{d^2P}{dr^2}$ o.e. or accept e.g. P'' BUT $\dfrac{d^2y}{dx^2}$ used in their conclusion is A0