Question	Scheme	Marks	AOs
16 (a)	$R=\int_{1}^{\sqrt{3}}\frac{1}{x^2\sqrt{4-x^2}}\,dx$		
	$\int\frac{1}{x^2\sqrt{4-x^2}}\,dx=\int\frac{1}{4\sin^2u\sqrt{4-4\sin^2u}}2\cos u\,(du)$	M1	3.1a
	Uses $1-\sin^2u=\cos^2u \Rightarrow \sqrt{4-4\sin^2u}=2\cos u$	dM1	1.1b
	$=\int\frac{1}{4\sin^2u\times 2\cos u}2\cos u\,(du)=\int\frac{1}{4}\text{cosec}^2u\,du$	A1	2.1
	Deduces $a=\frac{\pi}{6}$ and $b=\frac{\pi}{3}$	B1	2.2a
		(4)	
(b)	$\int\frac{1}{4}\text{cosec}^2u\,(du)=-\frac{1}{4}\cot u\ \ (+c)$	B1ft	1.2
	$=\left[-\frac{1}{4}\cot u\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=-\frac{1}{4}\cot\frac{\pi}{3}+\frac{1}{4}\cot\frac{\pi}{6}$	M1	2.1
	$=\frac{1}{2\sqrt{3}}$ or $\frac{\sqrt{3}}{6}$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a)

M1: Proceeds to $\frac{1}{A\sin^2u\sqrt{\pm C\pm C\sin^2u}}\times B\cos u$ o.e. or may be implied by $\frac{\cos u}{D\sin^2u\cos u}$ o.e in terms of u only $A, B, C, D\neq 0$

Requires $dx\to\ldots\cos u\ (du)$ o.e. Condone the omission of the integral sign and du for this mark. Condone the $\pm B\cos u$ appearing after the du if present

It cannot be implied by $\frac{1}{D\sin^2u}$ or $\frac{\text{cosec}^2u}{D}$ where $D\neq 0$

dM1: Attempts to use $\pm1\pm\sin^2u=\pm\cos^2u$ to change $\sqrt{\pm C\pm C\sin^2u}$ to $p\cos u$ o.e. (Must be seen somewhere in their solution which may be part of side-workings)

May be seen as e.g. $\frac{1}{A\sin^2u\sqrt{C\cos^2u}}\times B\cos u$ or $\frac{1}{A\sin^2u\times F\cos u}\times B\cos u$

It is dependent on the previous method mark.

A1: $\int\frac{1}{4}\text{cosec}^2u\,du$ Ignore any limits or the absence of them for this mark.

Requires the integral sign and du. Do not isw. Must be seen in (a)

B1: Deduces $a=\frac{\pi}{6}$ and $b=\frac{\pi}{3}$ (in radians) May be with their final integral in the correct positions or stated separately which value is a and which value is b. Must be seen in (a).

Alternative part (a) (Further Maths)

M1: $u = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{M}{\sqrt{1-Nx^2}}$ It cannot be implied by $\frac{1}{D\sin^2 u}$ or $\frac{\operatorname{cosec}^2 u}{D}$, $D \neq 0$

dM1: Uses $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{\sqrt{4-x^2}}$ o.e , substitutes $x = 2\sin u$ and proceeds to $\frac{1}{4\sin^2 u}$ It is dependent on the previous method mark.

A1: $\int \frac{1}{4}\operatorname{cosec}^2 u \, \mathrm{d}u$ Ignore any limits or the absence of them for this mark.

Requires the integral sign and du. Do not isw. Must be seen in (a)

B1: Deduces $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$ (must be in radians)

May be with their final integral in the correct positions or stated separately which value is a and which value is b. Must be seen in (a).

(b) Condone *x* instead of *u* provided the appropriate limits are substituted into the function

B1ft: $k\operatorname{cosec}^2 u \to -k\cot u$ (may be left in terms of $k \neq 0$)

e.g. $-2\operatorname{cosec}^2 u \to 2\cot u$ is B1 (look for the sign to change as well)

Note some candidates may prefer to change

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{4}\operatorname{cosec}^2 u\,(\mathrm{d}u) = -\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{1}{4}\operatorname{cosec}^2 u\,(\mathrm{d}u) = \frac{1}{4}\cot u \quad (+c)$$ which scores B1ft

M1: Uses the limits $\frac{\pi}{6}$ and $\frac{\pi}{3}$ either way round (or condone use of $30°$ and $60°$) in an expression of the form $\pm q\cot u$ and subtracts (either way round).

Allow $q = 1$. May be implied by their final answer provided B1ft has been scored.

May write as e.g. $-\frac{1}{4}\frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} + \frac{1}{4}\frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$

Do not condone a sign slip here e.g. $-\frac{1}{4}\cot\frac{\pi}{3} - \frac{1}{4}\cot\frac{\pi}{6}$ is M0.

The expression is sufficient but if just a value is stated following integration to the required form then you may need to check this on your calculator.

If no algebraic integration is seen then M0

If only decimal values are seen then M0

A1: $\frac{1}{2\sqrt{3}}$ or $\frac{\sqrt{3}}{6}$ provided the previous two marks have been scored.

Note that incorrect integration e.g. $\int k\operatorname{cosec}^2 u\,\mathrm{d}u = k\cot u \to \frac{1}{2\sqrt{3}}$ scores B0ftM1A0