

8. A student was asked to prove that the square of any number can be expressed in the form $5n$ or $5n \pm 1$ where $n \in \mathbb{N}$

The start of the student's proof is shown in the box below.

Let $m = 5k$

$$\text{Consider } m^2 = (5k)^2 = 25k^2 = 5 \times 5k^2 = 5n \quad \text{where } n = 5k^2 \quad \checkmark$$

Let $m = 5k + 1$

$$\begin{aligned} \text{Consider } m^2 &= (5k + 1)^2 \\ &= 25k^2 + 10k + 1 = 5(5k^2 + 2k) + 1 = 5n + 1 \quad \text{where } n = 5k^2 + 2k \quad \checkmark \end{aligned}$$

Let $m = 5k + 2$

$$\begin{aligned} \text{Consider } m^2 &= (5k + 2)^2 \\ &= 25k^2 + 10k + 4 = 5(5k^2 + 2k + 1) - 1 = 5n - 1 \quad \text{where } n = 5k^2 + 2k + 1 \quad \checkmark \end{aligned}$$

- (a) Identify and correct an algebraic error in the box above.

(1)

- (b) Show the calculations and statements that are required to complete the proof.

(4)