15.

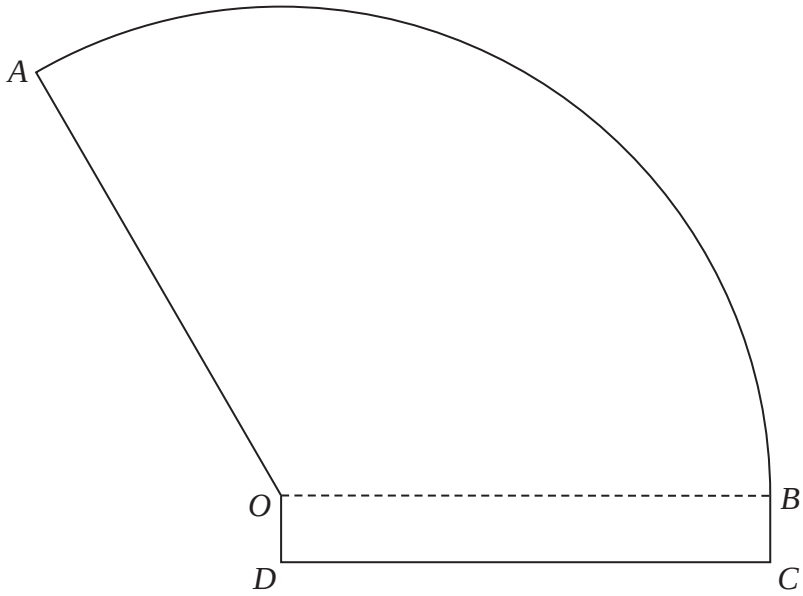

Figure 4

Figure 4 shows the plan view for the design of a stage.

The shape of this design consists of a sector of a circle AOB joined to a rectangle $OBCD$.

Given that

- the radius of the sector is r metres and angle AOB is θ radians
- the length and width of the rectangle are r metres and $\frac{1}{10}r$ metres respectively
- the total area of the stage is $240\,\text{m}^2$

(a) show that the perimeter of the stage, P metres, is given by

$$P = 2r + \frac{480}{r}$$

You must make your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which P has a stationary value.

(3)

(c) Prove, by further differentiation, that this value of r gives the minimum perimeter of the stage.

(2)