

1(a)	Suppose that $\log 7$ is rational $\log 7 = \frac{a}{b}$, where a and b are positive irrational numbers Begin process to rearrange e.g. $7 = 10^{\frac{a}{b}}$ $7^b = 10^a$ The integer powers of 7 are all odd, while the integer powers of 10 are even, hence $\log 7$ is irrational by contradiction	M1 M1 M1 M1 B1 [5]
1(b)	Suppose that $\frac{x+a}{\sqrt{x^2+a^2}} - \frac{x+b}{\sqrt{x^2+b^2}} \leq 0$ $\frac{x+a}{\sqrt{x^2+a^2}} \leq \frac{x+b}{\sqrt{x^2+b^2}}$ Since both sides are positive, $\frac{(x+a)^2}{x^2+a^2} \leq \frac{(x+b)^2}{x^2+b^2}$ $\frac{x^2+2ax+a^2}{x^2+a^2} \leq \frac{x^2+2bx+b^2}{x^2+b^2}$ $1 + \frac{2ax}{x^2+a^2} \leq 1 + \frac{2bx}{x^2+b^2}$ $\frac{2ax}{x^2+a^2} \leq \frac{2bx}{x^2+b^2}$ Since $2x > 0$ $\frac{a}{x^2+a^2} \leq \frac{b}{x^2+b^2}$ Since both denominators are positive, $a(x^2+b^2) \leq b(x^2+a^2)$ $ax^2+ab^2 \leq bx^2+ba^2$ $x^2(a-b) - ab(a-b) \leq 0$ $(a-b)(x^2-ab) \leq 0$ But $a > b \Rightarrow a-b > 0$ and $x^2 > ab \Rightarrow x^2-ab > 0$ Thus $(a-b)(x^2-ab) > 0$ Hence, proof by contradiction	M1 M1 M1 M1 M1 M1 M1 A1 [8]
1(c)	$P = (2^q - 1)^2 - 1 = 2^{2q} - 2^{q+1} + 1 - 1 = 2^{2q} - 2^{q+1}$ $P = 2^{q+1}(2^{q-1} - 1)$ Hence 2^{q+1} is a factor of P.	M1 A1 [2] [15 marks]