

$$A(20,18,-12) \ B(12,-6,20) \ C(4,y,z)$$

$$\overrightarrow{AC} = (4,y,z) - (20,18,-12) = (-16,y-18,z+12)$$

$$\overrightarrow{BC} = (4,y,z) - (12,-6,20) = (-8,y+6,z-20)$$

$$|\overrightarrow{AC}| = \sqrt{(-16)^2 + (y-18)^2 + (z+12)^2} = 24$$

$$y^2 - 36y + z^2 + 24z = -148 \quad (*)$$

$$|\overrightarrow{BC}| = \sqrt{(-8)^2 + (y+6)^2 + (z-20)^2} = 24$$

$$y^2 + 12y + z^2 - 40z = 76 \quad (**)$$

(*) subtract (**):

$$-48y + 64z = -224$$

$$4z + 14 = 3y$$

$$9(y^2 + 12y + z^2 - 40z) = 9(76)$$

$$9y^2 + 108y + 9z^2 - 360z = 684$$

$$(4z + 14)^2 + 36(4z + 14) + 9z^2 - 360z = 684$$

$$25z^2 - 104z + 16 = 0$$

$$(25z - 4)(z - 4) = 0$$

$$z = 4 \text{ or } \frac{4}{25}$$

M1

M1

M1

M1

M1

M1

M1

M1

M1 Students
may also
eliminate z

A1

10 cont.

$z = 4$:

$$4(4) + 14 = 3y \Rightarrow y = 10$$

$(4, 10, 4)$

$z = \frac{4}{25}$:

$$4\left(\frac{4}{25}\right) + 14 = 3y \Rightarrow y = \frac{122}{25}$$

$\left(4, \frac{122}{25}, \frac{4}{25}\right)$

A1

A1

[12]

[12 marks]