Question	Scheme	Marks	AOs
7(a)	$\left(\text{Area } R_1 =\right) \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ oe OR $\left(\text{Area } R_2 =\right) \frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)$ oe e.g. $\frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin\theta$	B1	2.2a
	$\left(\text{Area } R_1 =\right) \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ oe and $\left(\text{Area } R_2 =\right) \frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)$ oe	B1	2.2a
	$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = 2\left(\frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)\right)$ oe	M1	1.1b
	$(\theta - \sin\theta = 2\pi - 2\theta - 2\sin\theta)$ $\Rightarrow \sin\theta + 3\theta - 2\pi = 0$	A1	2.1
		(4)	
(b)	$\mathrm{f}'(\theta) = \cos\theta + 3$	B1ft	1.1b
	$\theta_1 = 1.3 \Rightarrow \theta_2 = 1.3 - \frac{\mathrm{f}(1.3)}{\mathrm{f}'(1.3)} = 1.3 - \frac{\sin(1.3) + 3(1.3) - 2\pi}{\cos(1.3) + 3}$	M1	1.1b
	$(\theta_2 =)$ awrt 1.734	A1	1.1b
		(3)	
			(7 marks)

Notes

(a) **Note in the expressions for R_1 or R_2 either of $\sin(\pi-\theta)$ or $\sin\theta$ may be used as they are equivalent. Condone missing trailing brackets throughout.**
Do not penalise the use of or switching between $\sin(180-\theta) \Leftrightarrow \sin(\pi-\theta)$

B1: A correct expression for **one** of the segments seen at least once. May be seen within an equation. Look for equivalent expressions e.g.

$$\left(\text{Area } R_1 =\right)\frac{1}{2}r^2\left(\theta - \sin(\pi-\theta)\right) \text{ or } \left(\text{Area } R_2 =\right)\frac{1}{2}r^2\left(\pi - \theta - \sin\theta\right)$$

If converting the angle in radians to degrees for the sector(s) look for e.g.

$$\left(\text{Area } R_1 =\right)\frac{\left(\frac{\theta}{\pi}\times 180\right)}{360}\pi r^2 - \frac{1}{2}r^2\sin\theta \quad \left(\text{Area } R_2 =\right)\frac{\left(180 - \frac{\theta}{\pi}\times 180\right)}{360}\pi r^2 - \frac{1}{2}r^2\sin(\pi-\theta)$$

B1: Correct expressions for **both** segments seen at least once. May see equivalent correct expressions. See note for 1st B1. May be seen within an equation. **Note they cannot just find R_2 (or R_1) using $R_1 = 2R_2$ to score this mark.**

M1: Correctly forms an equation using $R_1 = 2R_2$ where their areas are of the form $\left(\text{Area } R_1 =\right)\alpha r^2(\pm\theta \pm \sin\theta)$ oe and $\left(\text{Area } R_2 =\right) \beta r^2\left(\pm\pi \pm \theta \pm \sin\theta\right)$ oe

Note they may have already cancelled through by r^2 when they form the equation. Do not condone slips if they do not multiply through by 2 correctly oe for each term (or other similar methods). Invisible brackets would need to be implied by further work to ensure the correct equation was formed for their areas.

May alternatively form the equation $R_1 = \dfrac{2}{3}\left(R_1 + R_2\right)$ oe

e.g. $\dfrac{1}{2}r^2\theta - \dfrac{1}{2}r^2\sin\theta = \dfrac{2}{3}\left(\dfrac{1}{2}r^2\theta - \dfrac{1}{2}r^2\sin\theta + \dfrac{1}{2}r^2\left(\pi - \theta\right) - \dfrac{1}{2}r^2\sin\left(\pi - \theta\right)\right)$ oe

A1: $\sin\theta + 3\theta - 2\pi = 0$ oe with all terms on one side of the equation and not seen to come from an incorrect method. Must see $= 0$. All previous marks must have been scored.

Alt(a) Using the full semi-circle

B1: See main scheme notes

B1: Correct area of semi-circle $\dfrac{\pi r^2}{2}$ **and** the combined area of both triangles (may be separate) $\dfrac{1}{2}r^2\sin\theta + \dfrac{1}{2}r^2\sin\left(\pi - \theta\right)$ oe e.g. $r^2\sin\theta$

M1: Uses $R_1 = 2R_2$ to correctly form the equation $R_1 + R_2 = 3R_2$ oe **OR** $R_1 + R_2 = \dfrac{3}{2}R_1$ oe

where their individual areas are of the form $\left(\text{Area } R_1 =\right)\alpha r^2(\pm\theta \pm \sin\theta)$ oe or $\left(\text{Area } R_2 =\right) \beta r^2\left(\pm\pi \pm \theta \pm \sin\theta\right)$ and $R_1 + R_2 = \lambda\pi r^2 \pm \gamma r^2\left(\pm\sin\theta \pm \sin\left(\pi - \theta\right)\right)$ oe

Using R_1 e.g. $\dfrac{\pi r^2}{2} - \dfrac{1}{2}r^2\sin\theta - \dfrac{1}{2}r^2\sin\left(\pi - \theta\right) = \dfrac{3}{2}\left(\dfrac{1}{2}r^2\theta - \dfrac{1}{2}r^2\sin\theta\right)$ oe

Using R_2 e.g. $\dfrac{\pi r^2}{2} - \dfrac{1}{2}r^2\sin\theta - \dfrac{1}{2}r^2\sin\left(\pi - \theta\right) = 3\left(\dfrac{1}{2}r^2\left(\pi - \theta\right) - \dfrac{1}{2}r^2\sin\left(\pi - \theta\right)\right)$ oe

Note they may have already cancelled through by r^2 when they form the equation. Condone slips in simplifying their expressions for the areas before forming the equation.

A1: See main scheme notes

(b) Allow new values for *p* and *q* to be stated or used in part (b) if neither are found in part (a)

B1ft: $\left(\mathrm{f}'\left(\theta\right) =\right)\cos\theta + 3$ oe which may be unsimplified. e.g. $\mathrm{f}'(\theta) = -\cos\theta - 2\cos(\pi - \theta) + 3$

May be embedded in the N-R formula. Follow through on their *p* and *q* to achieve $p\cos\theta + q$ oe which may be unsimplified and allow if still in terms of *p* and *q*

M1: Attempts $1.3 - \dfrac{p\sin 1.3 + 1.3q - 2\pi}{\alpha\cos 1.3 + \beta}$ oe using their *p* and *q* (both non-zero) and $\alpha, \beta \neq 0$

The value 1.3 embedded in the correct places is sufficient for this mark and allow this mark to be scored if still in terms of *p* and *q* i.e. $\alpha = p$ and $\beta = q$

Note: Just stating $1.3-\dfrac{f(1.3)}{f'(1.3)}=...$ is M0 unless implied by further work

e.g. $f\left(1.3\right)$ and $f'\left(1.3\right)$ e.g. $1.3-\dfrac{\text{"}-1.419627122\text{"}}{\text{"}3.267498829\text{"}}=...$ **or** by their final answer

Must be a correct N-R formula used. You may need to check their values with accuracy of at least 3sf or allow truncated to 2dp using their *p* and *q*.

Allow if attempted in degrees. f(1.3) = -2.360497974 and $f'(1.3)=3.999742609$ and gives $\theta=1.890162469$ (again look for 3sf or allow truncated to 2dp)

Note that the full N-R calculator display is 1.734469053

The value of θ is approximately 1.7674890… and scores M0A0 without evidence of using the N-R formula to imply the M mark

A1: awrt 1.734 Ignore any subsequent iterations. Correct ans. with no incorrect working M1A1

Accept 1.7345 (may have rounded to 4dp) isw