Question	Scheme	Marks	AOs
9(a)	e.g. LHS: $(\cos\theta+\sin\theta)(\operatorname{cosec}\theta-\sec\theta)\equiv\dfrac{\cos\theta}{\sin\theta}-1+1-\dfrac{\sin\theta}{\cos\theta}$	M1	1.1b
	$\equiv\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}\equiv\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$ OR $\equiv\dfrac{1}{\tan\theta}-\tan\theta\equiv\dfrac{1-\tan^2\theta}{\tan\theta}$	dM1	3.1a
	$\equiv 2\cot 2\theta$	A1	2.1
		(3)	
Alt(a)	e.g. RHS: $k\cot 2\theta=\dfrac{k}{\tan 2\theta}\equiv\dfrac{k\cos 2\theta}{\sin 2\theta}\equiv\dfrac{k(\cos^2\theta-\sin^2\theta)}{2\sin\theta\cos\theta}$	M1	1.1b
	$\equiv\dfrac{k}{2}\dfrac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\sin\theta\cos\theta}\equiv\dfrac{k}{2}(\cos\theta+\sin\theta)(\operatorname{cosec}\theta-\sec\theta)$	dM1	3.1a
	$\Rightarrow k=2$	A1	2.1
		(3)	
(b)	$5(\cos x+\sin x)(\operatorname{cosec} x-\sec x)=4\operatorname{cosec}^2 2x$ $\Rightarrow 5\times 2\cot 2x=4\operatorname{cosec}^2 2x$	B1ft	1.1b
	e.g. $10\cot 2x=4\operatorname{cosec}^2 2x\Rightarrow 10\cot 2x=4(1+\cot^2 2x)$ $\Rightarrow 2\cot^2 2x-5\cot 2x+2=0\Rightarrow\cot 2x=2,\dfrac{1}{2}$ or e.g. $10\cot 2x=4\operatorname{cosec}^2 2x\Rightarrow\dfrac{10\cos 2x}{\sin 2x}=\dfrac{4}{\sin^2 2x}$ $\Rightarrow 5\cos 2x\sin^2 2x=2\sin 2x\Rightarrow\sin 2x(5\sin 2x\cos 2x-2)=0$ $\Rightarrow 5\sin 2x\cos 2x=2\Rightarrow\sin 4x=0.8$	M1	3.1a
	$\cot 2x=2,\dfrac{1}{2}\Rightarrow x=...$ or e.g. $\sin 4x=0.8\Rightarrow x=...$	dM1	2.1
	$(x=)$ awrt $-76.7°$, awrt $-58.3°$, awrt $13.3°$, awrt $31.7°$	A1A1	1.1b 1.1b
		(5)	
			(8 marks)

Notes

Condone a complete proof entirely in *x* (or another variable) instead of θ

Condone notational errors and missing variables in their working provided the intention is clear and only penalise other errors on the final A mark.

(a) e.g. working from the LHS

M1: Attempts to expand the brackets and uses both $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$ and $\sec\theta=\dfrac{1}{\cos\theta}$

Condone sign slips.

May see $\operatorname{cosec}\theta-\sec\theta=\dfrac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}\Rightarrow(\cos\theta+\sin\theta)\dfrac{(\cos\theta-\sin\theta)}{\sin\theta\cos\theta}=\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}$

Also allow this to be implied by e.g. $\left(\cos\theta+\sin\theta\right)(\operatorname{cosec}\theta-\sec\theta)=\pm\cot\theta\pm\tan\theta$

dM1: **EITHER** uses both $\cos^2\theta-\sin^2\theta\equiv\cos 2\theta$ and $\sin 2\theta\equiv 2\sin\theta\cos\theta$

OR uses $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and simplifies to a single fraction in $\tan\theta$ Dep. on previous M

A1: Achieves $2\cot 2\theta$ with all previous marks scored and no genuine errors including signs, manipulation and mixed variables.

Alt(a) e.g. working from the RHS

M1: Either uses both $\cos^2\theta-\sin^2\theta\equiv\cos 2\theta$ and $\sin 2\theta\equiv 2\sin\theta\cos\theta$ **or** uses $\tan 2\theta\equiv\dfrac{2\tan\theta}{1-\tan^2\theta}$

dM1: Uses the difference of two squares, and both $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$ and $\sec\theta=\dfrac{1}{\cos\theta}$

It is dependent on the previous method mark. Condone sign slips.

A1: Proceeds from $\dfrac{k}{2}\left(\cos\theta+\sin\theta\right)\left(\operatorname{cosec}\theta-\sec\theta\right)$ to deduce that $k=2$ with all previous marks scored and no genuine errors including signs, manipulation and mixed variables.

Note if they work from both sides they are likely to score the two method marks from each of M1 in the main scheme notes and alternative method notes. The final A will be scored for reaching a point where they can deduce that $k=2$

Score in the order which maximises the marks.

e.g.

M1: LHS: $\operatorname{cosec}\theta-\sec\theta=\dfrac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}\Rightarrow\left(\cos\theta+\sin\theta\right)\dfrac{\left(\cos\theta-\sin\theta\right)}{\sin\theta\cos\theta}=\cot\theta-\tan\theta$

(main scheme 1st M1)

dM1: RHS: $k\cot 2\theta=k\dfrac{1-\tan^2\theta}{2\tan\theta}=\dfrac{k}{2}\cot\theta-\dfrac{k}{2}\tan\theta$ (alternative method 1st M1)

A1: Achieves two similar statements from which $1=\dfrac{k}{2}\Rightarrow k=2$ with all previous marks scored and no genuine errors including signs, manipulation and mixed variables.

(b) May work in a different variable which is acceptable.

Condone mixed variables provided it is recovered in later work

B1ft: Deduces the correct equation using the result from part (a). Follow through their k and allow if in terms of k. i.e. $5\times k\cot 2x=4\operatorname{cosec}^2 2x$. Allow a restart but they would need to reach the same stage.

M1: Requires:

- EITHER uses $\pm 1\pm\cot^2 2x=\pm\operatorname{cosec}^2 2x$ to obtain **and solve** a 3TQ in $\cot 2x$. They can solve the 3TQ by factorising (may factorise for an equivalent quadratic equation which is acceptable), completing the square, formula (usual rules apply) or via a calculator. If just the roots are stated then they must **both** be correct to 2sf for their 3TQ. You may need to check this. This mark cannot be implied by proceeding from a 3TQ to an angle with no intermediate working. May multiply through by $\tan^2 2x$ and solve a 3TQ in $\tan 2x$.

- OR e.g. converts to $\sin 2x$ and $\cos 2x$ and uses $\sin 4x = 2\sin 2x\cos 2x$ to obtain an equation of the form $\sin 4x = p$ oe where $|p| < 1$ Do not be concerned by cancelling both sides by $\sin 2x$ and losing the equation $\sin 2x = 0$ (0 is not a valid solution)

dM1: Correct order of operations for their equation to obtain **at least one value for *x***. It is dependent on the previous method mark. e.g. $\sin 4x = p \Rightarrow x = \frac{1}{4}\sin^{-1} p = ...$

If just a value is stated you may need to check this on your calculator.

A1: For any 2 of awrt $-77°$, awrt $-58°$, awrt $13°$, awrt $32°$

Also allow in radians (awrt -1.3, awrt -1.0, awrt 0.23, awrt 0.55)

A1: awrt $-76.7°$, awrt $-58.3°$, awrt $13.3°$, awrt $31.7°$ and no extras in the given range (ignore any found outside of the range). Note the common additional solution of $0°$ is A0.

The degrees symbol is not required. Answers in radians score A0