Question	Scheme	Marks	AOs
10(a)	$x=2 \Rightarrow y=\dfrac{14}{\sqrt{12+4}}=\dfrac{7}{2}$	B1	1.1b
	$y=\dfrac{7x}{\sqrt{3x^2+4}} \Rightarrow \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\dfrac{7\sqrt{3x^2+4}-7x\times\frac{1}{2}\left(3x^2+4\right)^{-\frac{1}{2}}\times 6x}{3x^2+4}$	M1 A1	2.1 1.1b
	$\dfrac{7\sqrt{3(2)^2+4}-7(2)\times\frac{1}{2}\left(3(2)^2+4\right)^{-\frac{1}{2}}\times 6(2)}{3(2)^2+4}=\dfrac{7}{16}$ $\Rightarrow y-\dfrac{7}{2}=-\dfrac{16}{7}(x-2)$	M1	2.1
	$\Rightarrow 32x+14y-113=0$	A1	1.1b
		(5)	
(b)	$y=\dfrac{113}{14}$	B1ft	1.1b
		(1)	
(c)	Trapezium area is $\dfrac{1}{2}\left(\dfrac{113}{14}+\dfrac{7}{2}\right)\times 2 \quad \left(=\dfrac{81}{7}\right)$	B1ft	2.2a
	$\displaystyle\int \dfrac{7x}{\sqrt{3x^2+4}}\,\mathrm{d}x=\dfrac{7}{3}\left(3x^2+4\right)^{\frac{1}{2}}(+c)$	M1 A1	1.1b 1.1b
	Area is $\dfrac{81}{7}-\left[\dfrac{7}{3}\left(3x^2+4\right)^{\frac{1}{2}}\right]_0^2=\dfrac{81}{7}-\left(\dfrac{7}{3}\times 4-\dfrac{7}{3}\times 2\right)=\dfrac{81}{7}-\dfrac{14}{3}$	M1	3.1a
	$=\dfrac{145}{21}$	A1	1.1b
		(5)	
			(11 marks)

Notes

Check the diagram for all parts. If there is a contradiction between answers, the answer in the main body of the script takes precedence.

(a)

B1: $y=\dfrac{7}{2}$ or may see e.g. $\left(2,\dfrac{7}{2}\right)$ oe

M1: Differentiates to achieve the form (which may be unsimplified)

Quotient rule: $\dfrac{A\sqrt{3x^2+4}-Bx^2\left(3x^2+4\right)^{-\frac{1}{2}}}{3x^2+4}$ oe $A, B>0$

Product rule: $C(3x^2+4)^{-\frac{1}{2}}-Dx^2(3x^2+4)^{-\frac{3}{2}}$ oe $C, D>0$

Implicit: $E(3x^2+4)^{\frac{1}{2}}\dfrac{\mathrm{d}y}{\mathrm{d}x}+Fxy(3x^2+4)^{-\frac{1}{2}}=G$ oe $E, F, G\neq 0$ (May see alternatives)

A1: $\dfrac{7\sqrt{3x^2+4}-7x\times\frac{1}{2}\left(3x^2+4\right)^{-\frac{1}{2}}\times 6x}{3x^2+4}$ oe which may be unsimplified or simplified

e.g. $7(3x^2+4)^{-\frac{1}{2}}-21x^2(3x^2+4)^{-\frac{3}{2}}$ or $28(3x^2+4)^{-\frac{3}{2}}$ or $(3x^2+4)^{\frac{1}{2}}\dfrac{dy}{dx}+3xy(3x^2+4)^{-\frac{1}{2}}=7$

Whilst not strictly dependent on the first M, there must have been some attempt to differentiate i.e. they cannot get a gradient from substituting $x=2$ into the original equation (it must be a changed function) and using the negative reciprocal of this as their *m*. If no method is shown to find the *y* coordinate then it must be $\frac{7}{2}$

M1: A complete method for the equation of the normal:

- attempts to find the gradient at *P* using their $\dfrac{dy}{dx}$. Condone arithmetical slips and miscopying provided the intention is clear. If just a value is stated you may need to check this is correct for their $\dfrac{dy}{dx}$. (Accept e.g. $x=2$, $\dfrac{dy}{dx}=\ldots$ without needing to check)
- uses the negative reciprocal to find the gradient of the normal at *P* (note they may have already manipulated their $\dfrac{dy}{dx}$ to $-\dfrac{dx}{dy}$. $m\to-\dfrac{1}{m}$) This must be a correct process in the method
- uses their gradient of the normal with $x = 2$ and their y coordinate to find the equation of the normal i.e. $y-"\frac{7}{2}"=-\dfrac{1}{"\frac{7}{16}"}(x-2)$ Condone a sign slip on the $y-"\frac{7}{2}"$

  If they use $y=mx+c$ they must proceed as far as $c = \ldots$

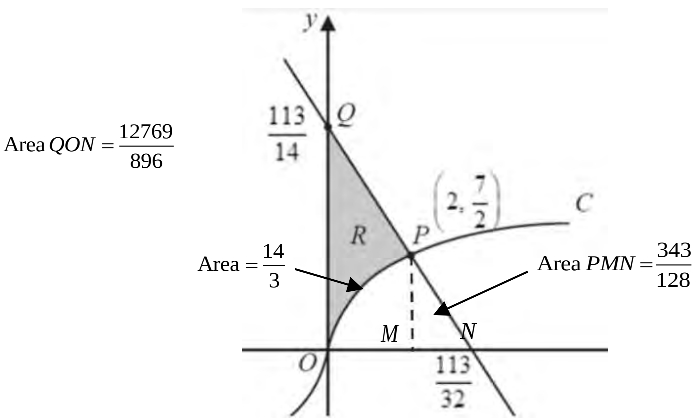
A1: $32x+14y-113=0$ Allow any integer multiple of this equation.

(b)

B1ft: $\dfrac{113}{14}$ or exact ft on their normal equation so allow for $-\dfrac{\text{their } c}{\text{their } b}$

Accept e.g. $\left(0,"\dfrac{113}{14}"\right)$ oe Also accept $-\dfrac{c}{b}$ or $\left(0,-\dfrac{c}{b}\right)$ if no equation was found in (a)

(c)



B1ft: Area of trapezium: $\frac{1}{2}\left("\frac{113}{14}"+"\frac{7}{2}"\right)\times 2$ oe e.g. $\frac{1}{2}\times"\frac{113}{14}"\times"\frac{113}{32}"-\frac{1}{2}\times\left("\frac{113}{32}"-2\right)\times"\frac{7}{2}"$

Follow through their part (b) and their y coordinate of P.
The expression is sufficient. The x coordinate of point N see the diagram above must be greater than 2 <u>if used</u> and the y coordinate of point Q must be greater than 0

Also allow if in terms of a, b and c where appropriate
e.g. $\frac{1}{2}\left(-\frac{c}{b}+"\frac{7}{2}"\right)\times 2$ oe e.g. $\frac{1}{2}\times\left(-\frac{c}{b}\right)\times\left(-\frac{c}{a}\right)-\frac{1}{2}\times\left(\left(-\frac{c}{a}\right)-2\right)\times"\frac{7}{2}"$ if no equation was found in (a)

Alternatively, may integrate the equation of l so look for

$$\int_0^2 "-\frac{16}{7}x+\frac{113}{14}"\,dx=\left["-\frac{8}{7}x^2+\frac{113}{14}x"\right]_0^2="-\frac{8}{7}"\times 2^2+"\frac{113}{14}"\times 2\left(="\frac{81}{7}"\right)$$

You would need to follow through on their l and they would need to have substituted into the correctly integrated expression.
isw if a correct expression is seen but they evaluate incorrectly.
If just a value is stated you may need to check that it is a correct follow through.

M1: Integrates via any method to obtain $D\left(3x^2+4\right)^{\frac{1}{2}}(+c)$ $D\neq 0$ May see attempts at the substitution method or even integration by parts but you just need to look for the correct form for their integrated expression to see that it is equivalent.
Note they may attempt

$$\int_0^2\left("-\frac{16}{7}x+\frac{113}{14}"-\frac{7x}{\sqrt{3x^2+4}}\right)dx$$ but ignore the integration on the straight line for this mark.

A1: $\frac{7}{3}\left(3x^2+4\right)^{\frac{1}{2}}(+c)$ oe which may be unsimplified. Look for the equivalent correct answer depending on their substitution e.g. using $u=3x^2+4\Rightarrow\frac{7}{3}\sqrt{u}\ (+c)$.

M1: Complete and correct strategy for the area of R. **Whilst not strictly dependent, there must have been some attempt to integrate to find the area under the curve, however poorly carried out.**

Look for area of their trapezium – area under the curve between $x=0$ and $x=2$ following an attempt to integrate where both areas are positive. <u>Condone subtracting either way round for this mark.</u>

e.g. $"\frac{81}{7}"-\left["\frac{7}{3}\left(3x^2+4\right)^{\frac{1}{2}}"\right]_0^2="\frac{81}{7}"-\left("\frac{7}{3}\times 4-\frac{7}{3}\times 2"\right)$

Look out for other longer methods incorporating additional areas which then need subtracting. The overall strategy needs to be correct but condone arithmetical slips provided the intention is clear.

Any areas which required integration should have had **both** limits substituted in and subtracted either way round (may be implied). If they have used the substitution method then the limits may be different for their integrated expression. e.g. using $u = 3x^2 + 4$ would require substituting in $u = 4$ and $u = 16$

A1: $\frac{145}{21}$ or exact equivalent e.g. $6\frac{19}{21}$ but do not allow a decimal