Question	Scheme	Marks	AOs
13(a)	$\dfrac{\sqrt{2}\cos\theta}{\sin\theta}=\dfrac{\sqrt{3}\cot\theta}{\sqrt{2}\cos\theta}$ or e.g. $\left(\sqrt{2}\cos\theta\right)^2=\sqrt{3}\cot\theta\sin\theta$	M1	3.1a
	$2\cos^2\theta=\sqrt{3}\cos\theta \Rightarrow \cos\theta=...\left(=\dfrac{\sqrt{3}}{2}\right)$	dM1	1.1b
	$r=\dfrac{\sqrt{2}\times\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ or e.g. $\dfrac{\sqrt{2}\cos\text{"}\frac{\pi}{6}\text{"}}{\sin\text{"}\frac{\pi}{6}\text{"}}$ or e.g. $\dfrac{\sqrt{3}\cot\text{"}\frac{\pi}{6}\text{"}}{\sqrt{2}\cos\text{"}\frac{\pi}{6}\text{"}}$	ddM1	2.1
	$r=\sqrt{6}$	A1	2.2a
		(4)	
(b)	e.g. $(u_1=)\dfrac{1}{\left(\sqrt{6}\right)^2}\sin\dfrac{\pi}{6}\left(=\dfrac{1}{12}\right)$ **OR** e.g. $(u_2=)\dfrac{\sin\text{"}\frac{\pi}{6}\text{"}}{\text{"}\sqrt{6}\text{"}}$	M1	3.1a
	$S_3=\dfrac{1}{12}+\dfrac{\sqrt{6}}{12}+\dfrac{6}{12}$ or $S_3=\dfrac{a\left(1-r^3\right)}{1-r}=\dfrac{1}{12}\left(\dfrac{1-\sqrt{6}^3}{1-\sqrt{6}}\right)$	dM1	2.1
	$=\dfrac{7+\sqrt{6}}{12}$ oe e.g. $\dfrac{7}{12}+\dfrac{\sqrt{6}}{12}$	A1	2.2a
		(3)	
Alt(b)	$u_3+u_4+u_5=\dfrac{1}{2}+\sqrt{2}\times\dfrac{\sqrt{3}}{2}+\sqrt{3}\times\dfrac{3}{\sqrt{3}}\left(=\dfrac{7+\sqrt{6}}{2}\right)$	M1	3.1a
	$\left(S_3=\dfrac{u_3+u_4+u_5}{r^2}=\right)\dfrac{\text{"}\frac{7+\sqrt{6}}{2}\text{"}}{\left(\text{"}\sqrt{6}\text{"}\right)^2}$	dM1	2.1
	$=\dfrac{7+\sqrt{6}}{12}$ oe e.g. $\dfrac{7}{12}+\dfrac{\sqrt{6}}{12}$	A1	2.2a
		(3)	
			(7 marks)

Notes

Mark (a) and (b) together and condone notational errors provided the intention is clear.
May work in degrees or radians – either is acceptable.

(a)

M1: Uses the 3 terms correctly to set up an equation in θ. Condone slips with the square roots but the trigonometric functions should be in the correct places. Give BOD where cos and cot may look similar when written.

$$\frac{\sqrt{2}\cos\theta}{\sin\theta}=\frac{\sqrt{3}\cot\theta}{\sqrt{2}\cos\theta} \text{ oe e.g. } \frac{\sqrt{2}\cos\theta}{\sin\theta}=\frac{\sqrt{3}\cos\theta}{\sqrt{2}\sin\theta\cos\theta}$$

May see e.g. $\sin\theta\dfrac{\sqrt{3}\cot\theta}{\sqrt{2}\cos\theta}=\sqrt{2}\cos\theta$

Condone a correct equation coming from $a=\sin\theta,\ \ ar=\sqrt{2}\cos\theta,\ ar^2=\sqrt{3}\cot\theta$

dM1: Proceeds from their equation and rearranges to find an **exact** value for $\cos\theta$ where $-1<\cos\theta<1$ and $\cos\theta\neq 0$

It is dependent on the previous method mark. May be implied by further work such as an **exact** value for θ in radians ($\frac{\pi}{6}$ (radians)) or in degrees ($30(°)$) but not just the final answer $\sqrt{6}$ Do not be concerned by the mechanics of the rearrangement.

Note for any further marks they must only <u>use</u> exact values

ddM1: Attempts e.g. $\dfrac{u_4}{u_3}=\dfrac{\sqrt{2}\cos\theta}{\sin\theta}$ using their exact value for $\cos\theta$ and the corresponding value for $\sin\theta=\sqrt{1-\left(\text{"}\dfrac{\sqrt{3}}{2}\text{"}\right)^2}$ and proceeds to find an **exact value** for *r*. It is dependent on both of the previous method marks.

Condone working with an exact value for $\cos\theta$ which would not give an exact value for θ provided they work with the exact values for $\cos\theta$ and $\sin\theta$ only. If they proceed to find an inexact value for θ and use this then this mark cannot be scored.

They must be attempting the division the correct way round.

If just an **exact** value of *r* is stated then you may need to check it is correct for their $\cos\theta$ (or θ if they have proceeded to find an **exact** θ and substituted in to find *r*)

A1: $\sqrt{6}$ cao (from a fully correct method)

Requires
- sight of a correct equation involving the 3 terms
- sight of either a correct value for $\cos\theta$ or θ

e.g. $\dfrac{\sqrt{2}\cos\theta}{\sin\theta}=\dfrac{\sqrt{3}}{\sqrt{2}\sin\theta}\Rightarrow\cos\theta=\dfrac{\sqrt{3}}{2}\Rightarrow(r=)\dfrac{\sqrt{2}\times\dfrac{\sqrt{3}}{2}}{\dfrac{1}{2}}=\sqrt{6}$ scores M1dM1ddM1A1

(b)

M1: A correct expression for the first term or second term which may be in terms of trigonometric functions (using u_3, u_4 or u_5) and/or uses their **exact** value for *r* from (a).

Note r may be expressed in terms of trigonometric functions but do not allow $r=\theta$ and r cannot clearly be an angle in radians or degrees e.g. $\frac{\pi}{6}$

e.g. $(u_1=)\frac{\sin"\frac{\pi}{6}"}{\left("\sqrt{6}"\right)^2}$ or $(u_1=)\ \frac{\sin\theta}{\left(\frac{\sqrt{2}\cos\theta}{\sin\theta}\right)^2}$ or $(u_2=)\frac{\sin"\frac{\pi}{6}"}{"\sqrt{6}"}$

May be implied by a correct **exact** value for a for their **exact** value for r
Condone if an exact value for r is not found in (a) and they just state a value for r then provided it is not clearly an angle in degrees or radians allow e.g. $r = 4$ $a=\frac{\sin\theta}{16}$ but not $r=\frac{\pi}{6}$ so $a=\frac{\sin\theta}{\left(\frac{\pi}{6}\right)^2}$ Note it is unlikely they will score any further marks unless they have chosen an exact value for r of either $\sqrt{6},\sqrt{2}$ or $\frac{\sqrt{6}}{3}$

dM1: Attempts to find the sum of the first 3 terms using their exact value for r from (a). It is dependent on the previous method mark.
The expression is sufficient which must be numerical and exact. **All trigonometric functions must have been dealt with for this mark.**

e.g. $(S_3=)\frac{1}{12}+\frac{\sqrt{6}}{12}+\frac{6}{12}$ or $\left(S_3=\frac{a\left(1-r^3\right)}{1-r}=\right)\frac{1}{12}\left(\frac{1-\sqrt{6}^3}{1-\sqrt{6}}\right)$

A1: $\frac{7+\sqrt{6}}{12}$ or exact equivalent in simplest form e.g. $\frac{7}{12}+\frac{\sqrt{6}}{12}$ (from a fully correct method)

Alt(b) $S_3=\frac{u_3+u_4+u_5}{r^2}$

M1: Adds the three terms in the question using their exact value for $\cos\theta$ where $-1<\cos\theta<1$ and $\cos\theta\neq 0$ and the corresponding value for $\sin\theta=\sqrt{1-\left("\frac{\sqrt{3}}{2}"\right)^2}$.

May be implied by $\frac{7+\sqrt{6}}{2}$ oe Condone if they just state a value for $\cos\theta$ if they did not achieve one in part (a) provided $-1<\cos\theta<1$ and $\cos\theta\neq 0$
Alternatively they may sum the three terms using their **exact value** for θ correctly (θ cannot be r). e.g. $\sin\left(\frac{\pi}{6}\right)+\sqrt{2}\cos\left(\frac{\pi}{6}\right)+\sqrt{3}\cot\left(\frac{\pi}{6}\right)$
Note they may use the sum formula using exact values for $\sin\theta$ and r
If the method does not show where these values come from then they need to be consistent with their exact value for $\cos\theta$

The expression is sufficient which must be numerical and **exact**.

dM1: Attempts to divide their sum of three terms by their exact value for r^2. It is dependent on the previous method mark. The expression is sufficient which must be numerical and **exact**. **All trigonometric functions must have been dealt with for this mark.**
If they did not achieve an exact value for r in (a) it is unlikely that their chosen value for $\cos\theta$ will lead to an exact value for r.

Note they may attempt $S_3 = \dfrac{u_3 + u_4 + u_5}{r^2}$ **or use the summation formula with** $r = \dfrac{1}{"\sqrt{6}"}$ **which would score M1dM1 at the same time. The numerical and exact expression would be sufficient to score both marks.**

A1: $\dfrac{7+\sqrt{6}}{12}$ or exact equivalent in simplest form e.g. $\dfrac{7}{12}+\dfrac{\sqrt{6}}{12}$ (from a fully correct method)

Beware of incorrect values for r leading to $\dfrac{7+\sqrt{6}}{2}$ or $\dfrac{7+\sqrt{6}}{12}$

e.g. $r = \dfrac{\sqrt{6}}{6} \Rightarrow u_3 + u_4 + u_5 \Rightarrow \dfrac{7+\sqrt{6}}{12}$ (M1dM0A0 via Alt method)

e.g. $r = \dfrac{\sqrt{6}}{6} \Rightarrow a = \dfrac{\sin\left(\dfrac{\pi}{6}\right)}{r^2} \Rightarrow S_3 = 3 + 3\dfrac{\sqrt{6}}{6} + 3\left(\dfrac{\sqrt{6}}{6}\right)^2 = \dfrac{7+\sqrt{6}}{2}$ (M1dM1A0 via Main scheme)