Question	Scheme	Marks	AOs
14(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=x(A-t)\Rightarrow\int\frac{1}{x}\mathrm{d}x=\int(A-t)\mathrm{d}t$ $\Rightarrow \ln x = At-\frac{t^2}{2}\ (+c)$ oe	M1 A1	3.1a 1.1b
	$t=0,\ x=0.3\Rightarrow \ln 0.3=c$ $t=4,\ x=21\Rightarrow \ln 21=4A-8+\ln 0.3$ $\Rightarrow A=...\left(\frac{8+\ln 70}{4}(=3.06212...)\right)$	M1	3.1b
	e.g. $\ln x=3.06t-\frac{t^2}{2}+\ln 0.3\Rightarrow x=0.3\mathrm{e}^{3.06t-\frac{t^2}{2}}$ or e.g. $x=0.3\mathrm{e}^{\left(\frac{8+\ln 70}{4}\right)t-\frac{t^2}{2}}$ or e.g. $x=0.3\mathrm{e}^{\text{awrt}\,3.06t-\frac{t^2}{2}}$	dM1 A1	2.1 3.3
		(5)	
(b)	Maximum occurs when $\frac{\mathrm{d}x}{\mathrm{d}t}=0\Rightarrow t=A$	B1ft	2.2a
	$x=0.3\mathrm{e}^{3.06\times3.06-\frac{3.06^2}{2}}=...$	M1	3.4
	$(x=)$ awrt 32.6 (mg L^{-1}) or awrt 32.4 (mg L^{-1})	A1	3.2a
		(3)	
(c)	$0.1=0.3\mathrm{e}^{\left(\frac{8+\ln 70}{4}\right)t-\frac{t^2}{2}}\Rightarrow\frac{1}{3}=\mathrm{e}^{\left(\frac{8+\ln 70}{4}\right)t-\frac{t^2}{2}}\Rightarrow\left(\frac{8+\ln 70}{4}\right)t-\frac{t^2}{2}=-\ln 3$ $\Rightarrow\frac{t^2}{2}-3.06t-\ln 3=0$	M1	3.1b
	$\frac{t^2}{2}-3.06t-\ln 3=0\Rightarrow t=...$	dM1	3.4
	$(T=)$ awrt 6.46	A1	2.3
		(3)	
			(11 marks)

Notes

(a)

M1: Separates the variables and integrates to the form $\ln x = At\pm\text{"}\frac{1}{2}\text{"}t^2\ (+c)$ oe

e.g. $\ln x=\left(A\pm\text{"}\frac{1}{2}\text{"}t\right)t\ (+c)$ or $\ln x=\text{"}\pm\frac{1}{2}\text{"}(A-t)^2\ (+c)$

A1: $\ln x = At-\frac{t^2}{2}\ (+c)$ oe

Note if there is no constant of integration in their expression then no further marks

can be scored in this part

M1: Requires:

- states or uses $t = 0$ and $x = 0.3$ to find their constant of integration.
- states or uses $t = 4$ and $x = 21$ to find A $(\neq 0)$

Both constants may be simplified or unsimplified. May be implied by their constants so may need to be checked. If working is seen they must be using their equation in x and t.

dM1: Proceeds from $\ln x = At \pm "\frac{1}{2}"t^2 + c$ oe to $x = e^{At \pm "\frac{1}{2}"t^2 + c}$ oe where A and c in their equation may be numerical if already found.

It is dependent on the <u>first</u> method mark only.

Condone sign slips and isw once a correct form is seen. Condone miscopying slips provided the intention is clear. Note that they may have done some rearranging in earlier work so you need to check the processing from when they have first integrated. Allow invisible brackets to be recovered or implied by further work.

A1: $x = 0.3e^{\left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2}}$ oe in the form $x = f(t)$ e.g. $x = \frac{3}{10}e^{2t + t\ln 70^{\frac{1}{4}} - \frac{t^2}{2}}$ or $x = 0.3e^{\left(\frac{8+\ln 70 - 2t}{4}\right)t}$

Allow e.g. $x = 0.3e^{\text{awrt}\,3.06t - \frac{t^2}{2}}$ or $x = \frac{3}{10}e^{2t + \text{awrt}\,1.06t - \frac{t^2}{2}}$ or $x = e^{\left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2} + \ln 0.3}$

Must be $x = \ldots$ and constants must be to 3sf or better where necessary. Condone the t appearing after the ln70 if seen and condone $-1.2(0)$ for $\ln 0.3$) Isw once a correct answer is seen.

(b)

B1ft: Deduces that the maximum occurs when $t = A$ or allow their numerical value for A to at least 3sf. Condone $A < 0$

M1: Uses their equation of the model to find a numerical value for x when $\frac{dx}{dt} = 0$

Condone if they do not deduce $t = A$ and attempt to differentiate their equation from part (a) and allowing them to state $\frac{dx}{dt} = 0,\ t = \ldots$ and use this value in their equation of the model to find a numerical value for x. In either approach condone the value of t to be negative. If the value for t is not seen in their equation of the model and just a value for x is stated you may need to check this on your calculator (or allow $t = "3.06" \Rightarrow x = \ldots$)

Note they may alternatively attempt to complete the square on

$x = "0.3e^{-\frac{t^2}{2} + 3.062t}" \Rightarrow x = "0.3e^{-\frac{1}{2}(t - 3.062)^2 + 4.6879..}"$ and deduce the maximum is when $t = "3.062"$ and proceeds to $x = "0.3e^{0 + 4.6879..}" = \ldots$

A1: awrt 32.6 (mg L^{-1}) ignore units. Condone awrt 32.4 for use of $A = 3.06$

(c) Allow use of *T* or *t* (or another letter)

M1: Sets their $f(t) = 0.1$ where $f(t)$ is of the form $\lambda e^{\alpha t^2 + \beta t + c}$ oe $\lambda, \alpha, \beta \neq 0$ (but c could be zero) and achieves a 3TQ equation in t which may be unsimplified. The = 0 may be implied by further work. May start with an earlier form of the equation from part (a) or proceed directly to this e.g. $\ln 0.1 = "3.06"t - \frac{t^2}{2} + \ln 0.3$ which scores M1

You do not need to be concerned by the mechanics of the rearrangement and condone reverting back from an incorrect form equation back to the required form.

dM1: Solves their 3TQ in t (or T) by any correct method including a calculator. If just the roots are stated with no working seen then you may need to check that the root(s) is/are correct for their 3TQ to 2sf. It is dependent on the previous method mark.

A1: awrt 6.46 with no other values and a correct 3TQ equation is seen which may be unsimplified. Condone use of rounded values provided an answer of awrt 6.46 is reached e.g. $\frac{t^2}{2} - "3.06"t + \ln 0.1 - \ln 0.3 (= 0)$