Question	Scheme	Marks	AOs
15	Assume that a and b are both odd so that $a=2m+1$ and $b=2n+1$ where $m,n\in\mathbb{N}, m\neq n$		
	$a^2+b^2=(2m+1)^2+(2n+1)^2$ e.g. $=4m^2+4m+4n^2+4n+2$ or e.g. $=2(2m^2+2m+2n^2+2n+1)$ $\Rightarrow$ which is even	M1 A1	1.1b 2.1
	e.g. So c^2 is even $\Rightarrow$ c is even $\Rightarrow c=2k$ say $\Rightarrow 2(2m^2+2m+2n^2+2n+1)=(2k)^2=4k^2$ $\Rightarrow 2m^2+2m+2n^2+2n+1=2k^2$	dM1	2.1
	As e.g. $2m^2+2m+2n^2+2n+1$ is odd and $2k^2$ is even there is a contradiction. Hence a and b cannot both be odd. *	A1*	2.4
		(4)	
			(4 marks)

Notes

M1: Substitutes the given algebraically distinct odd numbers into a^2+b^2, and expands. Terms do not need to be collected. Condone slips multiplying out including
$(2m+1)^2+(2n+1)^2=4m^2+1+4n^2+1$
Condone coefficient slips but they should have an expression of the form
$Am^2+Bm+Cn^2+Dn+E$ where $A,C,E\neq 0$
May be implied by their factorised form.

A1: Requires **both** a correct expression that is clearly even **AND**
they either need to state that the expression is even oe or deduce that c^2 is even
e.g. $4m^2+4m+4n^2+4n+2$ which is even
e.g. $2(2m^2+2m+2n^2+2n+1)$ hence c^2 is even (or condone so c is even)
e.g. $4(m^2+m+n^2+n)+2$ which is not odd

Condone dividing through by 2 to show that there are no fractions

Do not accept e.g. "$4m^2+4m+4n^2+4n+1+1$ which is even" as the expression is not in a form where it clearly is even – they would need +2 not +1+1 or e.g. factorised

dM1: A full argument leading to a contradiction
e.g. recognises that c^2 must be of the form $4k^2$ and then divides through by 2 (can use any letter other than a, b, c, m or n for k)

A1*: Fully correct proof with appropriate reasoning/explanation and conclusion.
Requires

- all previous marks scored
- deduces they have a contradiction because $2m^2+2m+2n^2+2n+1$ oe is **odd** and $2k^2$ is **even** oe i.e. they need to explain what the contradiction is between the two statements
- minimal conclusion e.g. hence proven, tick, QED

Do not penalise partial rephrasing of the statement that they were asked to prove in their conclusion. e.g. omitting "both" provided their statement does not clearly contradict the proof.

Note there are many alternative ways of proceeding to a contradiction which should be marked in the same way.

e.g. Considering multiples of 4

M1: See main scheme notes

A1: See main scheme notes

dM1: c^2 is even $\Rightarrow$ c is even $\Rightarrow c=2k$

The expression for a^2+b^2 must be written as $4\left(m^2+m+n^2+n\right)+2$ oe with the factor of 4 taken out and c^2 written as $4k^2$ or $4\times k^2$

Alternatively, they may divide through by 4

e.g. $4m^2+4m+4n^2+4n+2=\left(2k\right)^2=4k^2 \Rightarrow m^2+m+n^2+n+\dfrac{1}{2}=k^2$

(They can use any letter other than *a*, *b*, *c*, *m* or *n* for *k*)

A1*: Fully correct proof with appropriate reasoning/explanation and conclusion.
Requires

- all previous marks scored
- deduces they have a contradiction because $4\left(m^2+m+n^2+n\right)+2$ oe is two more than a multiple of 4 oe e.g. the right-hand side is divisible by 4 but the left-hand side is not i.e. they need to explain what the contradiction is between the two statements.
- minimal conclusion e.g. hence proven, tick, QED

Do not penalise partial rephrasing of the statement that they were asked to prove in their conclusion. e.g. omitting "both" provided their statement does not clearly contradict the proof.