**15.** A student attempts to answer the following question:

> Given that
>
> $$a^2 + b^2 = c^2 \quad a, b, c \in \mathbb{N} \quad a \neq b$$
>
> use algebra to prove by contradiction that $a$ and $b$ cannot **both** be odd.

The student starts the proof with:

> Assume that $a$ and $b$ are both odd so that
>
> $$a = 2m + 1 \text{ and } b = 2n + 1 \text{ where } m, n \in \mathbb{N} \quad m \neq n$$

The start of the student's proof is reprinted below.

Use detailed **algebraic** reasoning to complete the proof.

(You may assume without proof that if $k^2$ is even then $k$ is even, where $k \in \mathbb{N}$)

**(4)**

Assume that $a$ and $b$ are both odd so that

$$a = 2m + 1 \text{ and } b = 2n + 1 \text{ where } m, n \in \mathbb{N} \quad m \neq n$$