Question	Scheme	Marks	AOs
16(a)	Sets $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\frac{11}{2}$ into $1 = A(11-2P)+BP \Rightarrow A \text{ or } B$	M1	1.1b
	$\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\int \frac{22}{P(11-2P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$	B1	3.1a
	Uses (a) and attempts to integrate $\int \frac{2}{P} + \frac{4}{(11-2P)}\,\mathrm{d}P = t + c$	M1	1.1b
	$2\ln P - 2\ln(11-2P) = t + c$	A1	1.1b
	Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow \; c = (-2\ln 9)$	M1	3.1a
	Substitutes $P = 2 \Rightarrow t = 2\ln 2 + 2\ln 9 - 2\ln 7$	M1	3.1a
	Time $= 1.89$ years	A1	3.2a
		(6)	
(c)	Uses ln laws $2\ln P - 2\ln(11-2P) = t - 2\ln 9$ $\Rightarrow \ln\left(\frac{9P}{11-2P}\right) = \frac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow \left(\frac{9P}{11-2P}\right) = \mathrm{e}^{\frac{1}{2}t}$ $\Rightarrow 9P = (11-2P)\mathrm{e}^{\frac{1}{2}t}$ $\Rightarrow P = \mathrm{f}\left(\mathrm{e}^{\frac{1}{2}t}\right)$ or $\Rightarrow P = \mathrm{f}\left(\mathrm{e}^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P = \frac{11}{2 + 9\mathrm{e}^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$	A1	1.1b
		(3)	
		(12 marks)	

Question 16 continued

Notes:

(a)

B1: Sets $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{(11-2P)}$

M1: Substitutes $P=0$ or $P=\dfrac{11}{2}$ into $1 = A(11-2P)+BP \Rightarrow A \text{ or } B$

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\dfrac{1}{P(11-2P)} = \dfrac{1/11}{P} + \dfrac{2/11}{(11-2P)}$ or equivalent $\dfrac{1}{P(11-2P)} = \dfrac{1}{11P} + \dfrac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\displaystyle\int \frac{22}{P(11-2P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$ or equivalent

M1: Uses part (a) and $\displaystyle\int \frac{A}{P} + \frac{B}{(11-2P)}\mathrm{d}P = A\ln P \pm C\ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg $2\ln P - 2\ln\left(11-2P\right) = t+c$

M1: Substitutes $t = 0$ and $P = 1$ to find c

M1: Substitutes $P = 2$ to find t. This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2\ln P - 2\ln\left(11-2P\right) = t+c$ to $\ln\left(\dfrac{P}{11-2P}\right) = \dfrac{1}{2}t + d$ for their numerical 'c'

M1: Uses a correct method to get P in terms of $\mathrm{e}^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\dfrac{P}{11-2P}\right) = \dfrac{1}{2}t + d \Rightarrow \dfrac{P}{11-2P} = \mathrm{e}^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $\mathrm{e}^{-\frac{1}{2}t}$ For example

$\dfrac{P}{11-2P} = \mathrm{e}^{\frac{1}{2}t+d} \Rightarrow \dfrac{11-2P}{P} = \mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P} - 2 = \mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P} = 2 + \mathrm{e}^{-\left(\frac{1}{2}t+d\right)}$ followed by division

A1: Achieves the correct answer in the form required. $P = \dfrac{11}{2+9\mathrm{e}^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$ oe