

Question	Scheme	Marks	AOs
10 (a)	$x > \ln\left(\frac{4}{3}\right)$	B1	2.2a
		(1)	
(b)	Attempts to apply $\int y \frac{dx}{dt} dt$	M1	3.1a
	$\left\{ \int y \frac{dx}{dt} dt = \right\} = \int \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right) dt$	A1	1.1b
	$\frac{1}{(t+1)(t+2)} \equiv \frac{A}{t+1} + \frac{B}{t+2} \Rightarrow 1 \equiv A(t+2) + B(t+1)$	M1	3.1a
	$\{A=1, B=-1 \Rightarrow\}$ gives $\frac{1}{t+1} - \frac{1}{t+2}$	A1	1.1b
	$\left\{ \int \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt = \right\} \ln(t+1) - \ln(t+2)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln(t+1) - \ln(t+2)]_0^2 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right) *$	A1*	2.1
	(8)		
(b) Alt 1	Attempts to apply $\int y dx = \int \frac{1}{e^x - 2 + 1} dx = \int \frac{1}{e^x - 1} dx$, with a substitution of $u = e^x - 1$	M1	3.1a
	$\left\{ \int y dx \right\} = \int \left(\frac{1}{u}\right)\left(\frac{1}{u+1}\right) du$	A1	1.1b
	$\frac{1}{u(u+1)} \equiv \frac{A}{u} + \frac{B}{u+1} \Rightarrow 1 \equiv A(u+1) + Bu$	M1	3.1a
	$\{A=1, B=-1 \Rightarrow\}$ gives $\frac{1}{u} - \frac{1}{u+1}$	A1	1.1b
	$\left\{ \int \left(\frac{1}{u} - \frac{1}{u+1}\right) du = \right\} \ln u - \ln(u+1)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln u - \ln(u+1)]_1^3 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right) *$	A1 *	2.1
	(8)		

(9 marks)

Question	Scheme	Marks	AOs
10 (b) Alt 2	Attempts to apply $\int y dx = \int \frac{1}{e^x - 2 + 1} dx = \int \frac{1}{e^x - 1} dx$, with a substitution of $v = e^x$	M1	3.1a
	$\left\{ \int y dx \right\} = \int \left(\frac{1}{v-1} \right) \left(\frac{1}{v} \right) dv$	A1	1.1b
	$\frac{1}{(v-1)v} \equiv \frac{A}{v-1} + \frac{B}{v} \Rightarrow 1 \equiv Av + B(v-1)$	M1	3.1a
	$\{A = 1, B = -1 \Rightarrow\}$ gives $\frac{1}{(v-1)} - \frac{1}{v}$	A1	1.1b
	$\left\{ \int \left(\frac{1}{v-1} - \frac{1}{v} \right) dv = \right\} \ln(v-1) - \ln v$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln(v-1) - \ln v]_2^4 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln \left(\frac{(3)(2)}{4} \right) = \ln \left(\frac{6}{4} \right)$		
	$= \ln \left(\frac{3}{2} \right) *$	A1 *	2.1
		(8)	

Question 10 Notes:

(a)

B1: Uses $x = \ln(t+2)$ with $t > -\frac{2}{3}$ to deduce the correct domain, $x > \ln\left(\frac{4}{3}\right)$

(b)

M1: Attempts to solve the problem by either

- a parametric process or
- a Cartesian process with a substitution of either $u = e^x - 1$ or $v = e^x$

A1: Obtains

- $\int \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right) dt$ from a parametric approach
- $\int \left(\frac{1}{u}\right)\left(\frac{1}{u+1}\right) du$ from a Cartesian approach with $u = e^x - 1$
- $\int \left(\frac{1}{v-1}\right)\left(\frac{1}{v}\right) dv$ from a Cartesian approach with $v = e^x$

M1:

Applies a strategy of attempting to express either $\frac{1}{(t+1)(t+2)}$, $\frac{1}{u(u+1)}$ or $\frac{1}{(v-1)v}$ as partial fractions

A1: Correct partial fractions for their method

M1: Integrates to give either

- $\pm\alpha \ln(t+1) \pm \beta \ln(t+2)$
- $\pm\alpha \ln u \pm \beta \ln(u+1)$; $\alpha, \beta \neq 0$, where $u = e^x - 1$
- $\pm\alpha \ln(v-1) \pm \beta \ln v$; $\alpha, \beta \neq 0$, where $v = e^x$

A1: Correct integration for their method

M1: Either

- Parametric approach: Deduces and applies limits of 2 and 0 in t and subtracts the correct way round
- Cartesian approach: Deduces and applies limits of 3 and 1 in u , where $u = e^x - 1$, and subtracts the correct way round
- Cartesian approach: Deduces and applies limits of 4 and 2 in v , where $v = e^x$, and subtracts the correct way round

A1*: Correctly shows that the area of R is $\ln\left(\frac{3}{2}\right)$, with no errors seen in their working