Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16} (3+5r+2^r) = 131798$; (ii) $u_1, u_2, u_3,, : u_{n+1} = \frac{1}{u_n}, u_1 = \frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16} (3+5r+2^r) = \right\} \sum_{r=1}^{16} (3+5r) + \sum_{r=1}^{16} (2^r)$	M1	3.1a
	$16_{(2(2))+15(5))} + 2(2^{16}-1)$	M1	1.1b
	$= \frac{1}{2}(2(8)+13(3)) + \frac{1}{2-1}$	M1	1.1b
	= 728 + 131070 = 131798 *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16} (3+5r+2^r) = \right\} \sum_{r=1}^{16} 3 + \sum_{r=1}^{16} (5r) + \sum_{r=1}^{16} (2^r)$	M1	3.1a
	$(2 \times 16) + 16(2(5) + 15(5)) + 2(2^{16} - 1)$	M1	1.1b
	$= (3 \times 10) + \frac{1}{2}(2(3) + 15(3)) + \frac{1}{2-1}$	M1	1.1b
	= 48 + 680 + 131070 = 131798 *	A1*	2.1
		(4)	
		M1	3.1a
(i)	Sum = 10 + 17 + 26 + 39 + 60 + 97 + 166 + 299 + 560 + 1077 + 2106	M1	1.1b
way 3	+4159+8260+16457+32846+65619=131798*	M1	1.1b 2.1
		(4)	2.1
(ii)	$\left\{u_1=\frac{2}{3}\right\}, u_2=\frac{3}{2}, u_3=\frac{2}{3}, \dots$ (can be implied by later working)	M1	1.1b
	$\left\{\sum_{r=1}^{100} u_r = \right\} \ 50\left(\frac{2}{3}\right) + 50\left(\frac{3}{2}\right) \text{ or } 50\left(\frac{2}{3} + \frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3} \left(\text{or } 108\frac{1}{3} \text{ or } 108.3 \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6} \right)$	A1	1.1b
		(3)	
		(7	marks)

Notes for Question 4		
(i)		
M1:	Uses a correct methodical strategy to enable the given sum, $\sum_{r=1}^{16} (3+5r+2^r)$ to be found	
	Allow M1 for any of the following:	
	 expressing the given sum as either 	
	$\sum_{r=1}^{16} (3+5r) + \sum_{r=1}^{16} (2^r), \sum_{r=1}^{16} 3 + \sum_{r=1}^{16} (5r) + \sum_{r=1}^{16} (2^r) \text{ or } \sum_{r=1}^{16} 3 + 5\sum_{r=1}^{16} r + \sum_{r=1}^{16} (2^r)$	
	• attempting to find both $\sum_{r=1}^{r} (3+5r)$ and $\sum_{r=1}^{r} (2^r)$ separately	
	• (3×16) and attempting to find both $\sum_{r=1}^{16} (5r)$ and $\sum_{r=1}^{16} (2^r)$ separately	
M1:	Way 1: Correct method for finding the sum of an AP with $a = 8, d = 5, n = 16$	
	Way 2: (3×16) and a correct method for finding the sum of an AP	
M1:	Correct method for finding the sum of a GP with $a = 2, r = 2, n = 16$	
A1*:	For all steps fully shown (with correct formulae used) leading to 131798	
Note:	Way 1: Give 2 nd M1 for writing $\sum_{r=1}^{16} (3+5r)$ as $\frac{16}{2}(8+83)$	
Note:	Way 2: Give 2 nd M1 for writing $\sum_{r=1}^{16} 3 + \sum_{r=1}^{16} (5r)$ as $48 + \frac{16}{2}(5+80)$ or $48 + 680$	
Note:	Give 3 rd M1 for writing $\sum_{r=1}^{16} (2^r)$ as $\frac{2(1-2^{16})}{1-2}$ or $2(2^{16}-1)$ or $(2^{17}-2)$	
(i)		
Way 3		
M1:	At least 6 correct terms and 16 terms shown	
M1:	At least 10 correct terms (may not be 16 terms)	
M1:	At least 15 correct terms (may not be 16 terms)	
A1*:	All 16 terms correct and an indication that the sum is 131798	
(ii)		
M1:	For some indication that the next two terms of this sequence are $\frac{3}{2}, \frac{2}{3}$	
M1:	For deducing that the sum can be found by applying $50\left(\frac{2}{3}\right) + 50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3} + \frac{3}{2}\right)$, o.e.	
A1:	Obtains $\frac{325}{3}$ or $108\frac{1}{3}$ or 108.3 or an exact equivalent	
Note:	Allow 1 st M1 for $u_2 = \frac{3}{2}$ (or equivalent) and $u_3 = \frac{2}{3}$ (or equivalent)	
Note:	Allow 1 st M1 for the first 3 terms written as $\frac{2}{3}, \frac{3}{2}, \frac{2}{3}, \dots$	
Note:	Allow 1 st M1 for the 2 nd and 3 rd terms written as $\frac{3}{2}, \frac{2}{3}, \dots$ in the correct order	
Note:	Condone $\frac{2}{3}$ written as 0.66 or awrt 0.67 for the 1 st M1 mark	
Note:	Give A0 for 108.3 or 108.333 without reference to $\frac{325}{3}$ or $108\frac{1}{3}$ or 108.3	

Г