Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\left\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\right\}\ f'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \Rightarrow\right\}\ \{x_{n+1}\} = x_n - \dfrac{2x_n^{\ 3} + x_n^{\ 2} - 1}{6x_n^{\ 2} + 2x_n}$	M1	1.1b
	$= \dfrac{x_n(6x_n^{\ 2} + 2x_n) - (2x_n^{\ 3} + x_n^{\ 2} - 1)}{6x_n^{\ 2} + 2x_n} \Rightarrow x_{n+1} = \dfrac{4x_n^{\ 3} + x_n^{\ 2} + 1}{6x_n^{\ 2} + 2x_n}$ *	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\}\ x_2 = \dfrac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \dfrac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \dfrac{3}{4},\ x_3 = \dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

Notes for Question 5

(a)	
B1:	States that $f'(x) = 6x^2 + 2x$ or states that $f'(x_n) = 6x_n^{\ 2} + 2x_n$ (Condone $\dfrac{dy}{dx} = 6x^2 + 2x$)
M1:	Substitutes $f(x_n) = 2x_n^{\ 3} + x_n^{\ 2} - 1$ and their $f'(x_n)$ into $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $f'(x) = 6x^2 + 2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\{x_{n+1}\} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
Note:	Allow M1A1 for • $x_{n+1} = x - \dfrac{2x^3 + x^2 - 1}{6x^2 + 2x} = \dfrac{x(6x^2 + 2x) - (2x^3 + x^2 - 1)}{6x^2 + 2x} \Rightarrow x_{n+1} = \dfrac{4x_n^{\ 3} + x_n^{\ 2} + 1}{6x_n^{\ 2} + 2x_n}$
Note	Condone $x = x - \dfrac{2x^3 + x^2 - 1}{\text{"}6x^2 + 2x\text{"}}$ for M1
Note	Condone $x_n - \dfrac{2x_n^{\ 3} + x_n^{\ 2} - 1}{\text{"}6x_n^{\ 2} + 2x_n\text{"}}$ or $x - \dfrac{2x^3 + x^2 - 1}{\text{"}6x^2 + 2x\text{"}}$ (i.e. no $x_{n+1} = ...$) for M1
Note:	Give M0 for $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1} = 2x_n^{\ 3} + x_n^{\ 2} - 1 - \dfrac{2x_n^{\ 3} + x_n^{\ 2} - 1}{6x_n^{\ 2} + 2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*

Notes for Question 5 Continued	
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1=1$
A1:	$x_2=\frac{3}{4}$ and $x_3=\frac{2}{3}$
Note:	Condone $x_2=\frac{3}{4}$ and $x_3=$ awrt 0.667 for A1
Note:	Condone $\frac{3}{4}, \frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x=0$ • At $x=0$, $f'(x_1)=0$ • x_1 cannot be 0 • $6x^2+2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1=0$, $6x^2+2x=0$