Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}$ $\left\{= \frac{2\pm\sqrt{3}}{4} \text{ or } 0.933..., 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta-\alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ | $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$ | $\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ | e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
			(9 marks)

Question	Scheme		Marks	AOs
7	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) Alt 1	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta + \sin^2\theta = 1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4 + 20\cos\theta + 25\cos^2\theta$ $\Rightarrow 25(1-\cos^2\theta) = 4 + 20\cos\theta + 25\cos^2\theta$ **or** e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1-\sin^2\theta)$		M1	1.1b
	$50\cos^2\theta + 20\cos\theta - 21 = 0$	$50\sin^2\theta - 20\sin\theta - 21 = 0$		
	$\cos\theta = \dfrac{-20 \pm \sqrt{4600}}{100}$, o.e.	$\sin\theta = \dfrac{20 \pm \sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\dfrac{-2+\sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\dfrac{2+\sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Notes for Question 7

(i)	
B1:	For recalling that $\sec x = \dfrac{1}{\cos x}$
M1:	Correct strategy of • **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \lvert k\rvert \le 1,\ k \ne 0$ • **Way 2:** squaring both sides, applying $\cos^2 x + \sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \lvert k\rvert \le 1,\ k \ne 0$
dM1:	Uses the correct order of operations to find at least one value for x in either radians or degrees
A1:	Clear reasoning to achieve both $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ and no other values in the range $0 \le x < \dfrac{\pi}{2}$
Note:	Give dM1 for $\sin 2x = \dfrac{1}{2} \Rightarrow$ any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, 15°, 75°$, awrt 0.26 or awrt 1.3
Note:	Give special case, SC B1M0M0A0 for writing down any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, 15°$ or $75°$ with no working

Notes for Question 7 Continued	
(ii)	
M1:	See scheme
Note:	**Alternative strategy:** Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\cos(\theta+\alpha) = -2$, finds both R and α, and proceeds to $\cos(\theta+\alpha) = k$, $\lvert k \rvert < 1$, $k \neq 0$
M1:	Either • uses $R\sin(\theta-\alpha)$ to find the values of both R and α • attempts to apply $(5\sin\theta - 5\cos\theta)^2 = 2^2$, uses $\cos^2\theta + \sin^2\theta = 1$ and proceeds to find an equation of the form $\pm\lambda \pm \mu\sin 2\theta = \pm\beta$ or $\pm\mu\sin 2\theta = \pm\beta$; $\mu \neq 0$ • attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ and uses $\cos^2\theta + \sin^2\theta = 1$ to form an equation in $\cos\theta$ only or $\sin\theta$ only
A1:	For $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$, o.e., $\cos(\theta + 45°) = -\frac{2}{\sqrt{50}}$, o.e. or $\sin 2\theta = \frac{21}{25}$, o.e. or $\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e. or $\cos\theta =$ awrt 0.48, awrt -0.88 or $\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e., or $\sin\theta =$ awrt 0.88, awrt -0.48
Note:	$\sin(\theta - 45°)$, $\cos(\theta + 45°)$, $\sin 2\theta$ must be made the subject for A1
dM1:	**dependent on the first M mark** Uses the correct order of operations to find at least one value for x in either degrees or radians
Note:	dM1 can also be given for $\theta = 180° - \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ or $\theta = \frac{1}{2}\left(180° - \arcsin\left(\frac{21}{25}\right)\right)$
A1:	Clear reasoning to achieve both $\theta =$ awrt 61.4°, awrt 208.6° and no other values in the range $0 \leq \theta < 360°$
Note:	Give M0M0A0M0A0 for writing down any of $\theta =$ awrt 61.4°, awrt 208.6° with no working
Note:	<u>**Alternative solutions:**</u> (to be marked in the same way as Alt 1): • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5\tan\theta - 5 = 2\sec\theta \Rightarrow (5\tan\theta - 5)^2 = (2\sec\theta)^2$ $\Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4\sec^2\theta \Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4(1+\tan^2\theta)$ $\Rightarrow 21\tan^2\theta - 50\tan\theta + 21 = 0 \Rightarrow \tan\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5 - 5\cot\theta = 2\text{cosec}\,\theta \Rightarrow (5 - 5\cot\theta)^2 = (2\text{cosec}\,\theta)^2$ $\Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4\text{cosec}^2\theta \Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4(1+\cot^2\theta)$ $\Rightarrow 21\cot^2\theta - 50\cot\theta + 21 = 0 \Rightarrow \cot\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only