Question	Scheme	Marks	AOs
9	$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta+h) - \cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1 A1	1.1b 1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
			(5 marks)

Notes for Question 9

B1:	Gives the correct fraction such as $\frac{\cos(\theta+h) - \cos\theta}{h}$ or $\frac{\cos(\theta+\delta\theta) - \cos\theta}{\delta\theta}$ Allow $\frac{\cos(\theta+h) - \cos\theta}{(\theta+h) - \theta}$ o.e. **Note:** $\cos(\theta+h)$ or $\cos(\theta+\delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta+h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\frac{d}{d\theta}(\cos\theta) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of ***curve*** is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\frac{d}{d\theta}(\cos\theta) = -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $f'(\theta)$ where $f(\theta) = \cos\theta$ or $\frac{dy}{d\theta}$ where $y = \cos\theta$ used in place of $\frac{d}{d\theta}(\cos\theta)$

Notes for Question 9 Continued	
Note:	Condone x used in place of θ if this is done consistently
Note:	Give final A0 for • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim\limits_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • $\frac{\mathrm{d}}{\mathrm{d}\theta} = \ldots$ • Defining $\mathrm{f}(x) = \cos\theta$ and applying $\mathrm{f}'(x) = \ldots$ • $\frac{\mathrm{d}}{\mathrm{d}x}(\cos\theta)$
Note:	Give final A1 for a correct limiting argument in x, followed by $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ e.g. $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim\limits_{h \to 0}\left(-\frac{\sin h}{h}\sin x + \left(\frac{\cos h - 1}{h}\right)\cos x\right) = -1\sin x + 0\cos x = -\sin x$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Applying $h \to 0,\ \sin h \to h,\ \cos h \to 1$ to give e.g. $\lim\limits_{h \to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \left(\frac{\cos\theta(1) - \sin\theta(h) - \cos\theta}{h}\right) = \frac{-\sin\theta(h)}{h} = -\sin\theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim\limits_{h \to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \lim\limits_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right)$ $= \lim\limits_{h \to 0}\left(-(1)\sin\theta + 0\cos\theta\right) = -\sin\theta$. So for not removing $\lim\limits_{h \to 0}$ when the limit was taken is final A0
Note:	**<u>Alternative Method:</u>** Considers $\frac{\cos(\theta + h) - \cos(\theta - h)}{(\theta + h) - (\theta - h)}$ which simplifies to $\frac{-2\sin\theta\sin h}{2h}$