

Question	Scheme	Marks	AOs		
<b>10 (a)</b>	$\frac{dr}{dt} \propto \pm \frac{1}{r^2}$ or $\frac{dr}{dt} = \pm \frac{k}{r^2}$ (for $k$ or a numerical $k$ )	M1	3.3		
	$\int r^2 dr = \int \pm k dt \Rightarrow \dots$ (for $k$ or a numerical $k$ )	M1	2.1		
	$\frac{1}{3}r^3 = \pm kt \{+ c\}$	A1	1.1b		
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;"> <math>t = 0, r = 5</math> and <math>t = 4, r = 3</math>  gives <math>\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}</math>,  where <math>r</math>, in mm, is the radius  {of the mint} and <math>t</math>, in minutes, is  the time from when it {the mint}  was placed in the mouth </td> <td style="width: 50%; padding: 5px;"> <math>t = 0, r = 5</math> and <math>t = 240, r = 3</math>  gives <math>\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}</math>,  where <math>r</math>, in mm, is the radius  {of the mint} and <math>t</math>, in seconds, is  the time from when it {the mint}  was placed in the mouth </td> </tr> </table>	$t = 0, r = 5$ and $t = 4, r = 3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$ , where $r$ , in mm, is the radius {of the mint} and $t$ , in minutes, is the time from when it {the mint} was placed in the mouth	$t = 0, r = 5$ and $t = 240, r = 3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$ , where $r$ , in mm, is the radius {of the mint} and $t$ , in seconds, is the time from when it {the mint} was placed in the mouth	M1	3.1a
	$t = 0, r = 5$ and $t = 4, r = 3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$ , where $r$ , in mm, is the radius {of the mint} and $t$ , in minutes, is the time from when it {the mint} was placed in the mouth	$t = 0, r = 5$ and $t = 240, r = 3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$ , where $r$ , in mm, is the radius {of the mint} and $t$ , in seconds, is the time from when it {the mint} was placed in the mouth			
	A1	1.1b			
	<b>(5)</b>				
<b>(b)</b>	$r = 0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = \dots$	M1	3.4		
	time = 5 minutes 6 seconds	A1	1.1b		
		<b>(2)</b>			
<b>(c)</b>	<p>Suggests a suitable limitation of the model. E.g.</p> <ul style="list-style-type: none"> <li>• Model does not consider how the mint is sucked</li> <li>• Model does not consider whether the mint is bitten</li> <li>• Model is limited for times up to 5 minutes 6 seconds, o.e.</li> <li>• Not valid for times greater than 5 minutes 6 seconds, o.e.</li> <li>• Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked</li> <li>• The model indicates that the radius of the mint is negative after it dissolves</li> <li>• Model does not consider the temperature in the mouth</li> <li>• Model does not consider rate of saliva production</li> <li>• Mint could be swallowed before it dissolves in the mouth</li> </ul>	B1	3.5b		
		<b>(1)</b>			

**(8 marks)**

## Notes for Question 10

<b>(a)</b>	
<b>M1:</b>	Translates the description of the model into mathematics. See scheme.
<b>M1:</b>	Separates the variables of their differential equation which is in the form $\frac{dr}{dt} = f(r)$ and some attempt at integration. (e.g. attempts to integrate at least one side). e.g. $\int r^2 dr = \int \pm k dt$ and some attempt at integration. Condone the lack of integral signs
<b>Note:</b>	You can imply the M1 mark for $r^2 dr = -k dt \Rightarrow \frac{1}{3} r^3 = -kt$
<b>Note:</b>	A numerical value of $k$ (e.g. $k = \pm 1$ ) is allowed for the first two M marks
<b>A1:</b>	Correct integration to give $\frac{1}{3} r^3 = \pm kt$ with or without a constant of integration, $c$
<b>M1:</b>	For a complete process of using the boundary conditions to find both their unknown constants and finds an equation linking $r$ and $t$ So applies either <ul style="list-style-type: none"> <li><math>t = 0, r = 5</math> and <math>t = 4, r = 3</math>, or</li> <li><math>t = 0, r = 5</math> and <math>t = 240, r = 3</math>,</li> </ul> <i>on their integrated equation</i> to find their constants $k$ and $c$ <i>and</i> obtains an equation linking $r$ and $t$
<b>A1:</b>	Correct equation, with variables $r$ and $t$ fully defined including correct reference to units. <ul style="list-style-type: none"> <li><math>\frac{1}{3} r^3 = -\frac{49}{6} t + \frac{125}{3}</math>, {or an equivalent equation,} where <math>r</math>, in mm, is the radius {of the mint} and <math>t</math>, in minutes, is the time from when it {the mint} was placed in the mouth</li> <li><math>\frac{1}{3} r^3 = -\frac{49}{360} t + \frac{125}{3}</math>, {or an equivalent equation,} where <math>r</math>, in mm, is the radius {of the mint} and <math>t</math>, in seconds, is the time from when it {the mint} was placed in the mouth</li> </ul>
<b>Note:</b>	Allow correct equations such as <ul style="list-style-type: none"> <li>in minutes, <math>r = \sqrt[3]{\frac{250 - 49t}{2}}</math>, <math>r^3 = -\frac{49}{2} t + 125</math> or <math>t = \frac{250 - 2r^3}{49}</math></li> <li>in seconds, <math>r = \sqrt[3]{\frac{15000 - 49t}{120}}</math>, <math>r^3 = -\frac{49}{120} t + 125</math> or <math>t = \frac{15000 - 120r^3}{49}</math></li> </ul>
<b>Note:</b>	$t$ defined as “the time from the start” is not sufficient for the final A1
<b>(b)</b>	
<b>M1:</b>	Sets $r = 0$ in their part (a) equation which links $r$ with $t$ and rearranges to make $t = \dots$
<b>A1:</b>	5 minutes 6 seconds cao ( <b>Note:</b> 306 seconds with no reference to 5 minutes 6 seconds is A0)
<b>Note:</b>	Give M0 if their equation would solve to give a negative time or a negative time is found
<b>Note:</b>	You can mark part (a) and part (b) together
<b>(c)</b>	
<b>B1:</b>	See scheme
<b>Note:</b>	Do not accept by itself <ul style="list-style-type: none"> <li>mint may not dissolve at a constant rate</li> <li>rate of decrease of mint must be constant</li> <li><math>0 \leq t &lt; \frac{250}{49}</math>, <math>r \geq 0</math>; without any written explanation</li> <li>reference to a mint having <math>r &gt; 5</math></li> </ul>