

Question	Scheme	Marks	AOs
11	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using $-10x+10 \equiv B(1-2x) + C(x-3)$	A1	1.1b
		(4)	
(b)	$f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}$ { $= 3 + 4(x-3)^{-1} - 2(1-2x)^{-1}$ }; $x > 3$		
	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2} \left\{ = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2} \right\}$	M1 A1ft	2.1 1.1b
	Correct $f'(x)$ and as $(x-3)^2 > 0$ and $(1-2x)^2 > 0$ , then $f'(x) = -(+ve) - (+ve) < 0$ , so $f(x)$ is a decreasing function	A1	2.4
		(3)	

(7 marks)

**Notes for Question 11**

(a)	
M1:	Way 1: Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3)$ in a complete method to find values for $B$ and $C$ . Note: Allow one slip in copying $1+11x-6x^2$ Way 2: Uses a correct identity $-10x+10 \equiv B(1-2x) + C(x-3)$ (which has been found from long division) in a complete method to find values for $B$ and $C$
B1:	$A = 3$
M1:	Attempts to find the value of either $B$ or $C$ from their identity This can be achieved by <i>either</i> substituting values into their identity <i>or</i> by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	Way 1: Comparing terms: $x^2: -6 = -2A$ ; $x: 11 = 7A - 2B + C$ ; constant: $1 = -3A + B - 3C$ Way 1: Substituting: $x = 3: -20 = -5B \Rightarrow B = 4$ ; $x = \frac{1}{2}: 5 = -\frac{5}{2}C \Rightarrow C = -2$
Note:	Way 2: Comparing terms: $x: -10 = -2B + C$ ; constant: $10 = B - 3C$ Way 2: Substituting: $x = 3: -20 = -5B \Rightarrow B = 4$ ; $x = \frac{1}{2}: 5 = -\frac{5}{2}C \Rightarrow C = -2$

<b>Note:</b>	$A=3, B=4, C=-2$ from no working scores M1B1M1A1
<b>Note:</b>	The final A1 mark is effectively dependent upon both M marks

Notes for Question 11 Continued	
<b>(a) ctd</b>	
<b>Note:</b>	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4, C=-2$ will get 1 <sup>st</sup> M0, 2 <sup>nd</sup> M1, 1 <sup>st</sup> A0
<b>Note:</b>	<p><b>Way 1:</b> You can imply a correct identity <math>1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)</math></p> <p>from seeing <math>\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}</math></p>
<b>Note:</b>	<p><b>Way 2:</b> You can imply a correct identity <math>-10x+10 \equiv B(1-2x)+C(x-3)</math></p> <p>from seeing <math>\frac{-10x+10}{(x-3)(1-2x)} \equiv \frac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}</math></p>
<b>(b)</b>	
<b>M1:</b>	Differentiates to give $\{f'(x) = \} \pm \lambda(x-3)^{-2} \pm \mu(1-2x)^{-2}; \lambda, \mu \neq 0$
<b>A1ft:</b>	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ , which can be simplified or un-simplified
<b>Note:</b>	Allow A1ft for $f'(x) = -(their\ B)(x-3)^{-2} + (2)(their\ C)(1-2x)^{-2}; (their\ B), (their\ C) \neq 0$
<b>A1:</b>	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $f'(x) = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2}$ <b>and</b> a correct explanation e.g. $f'(x) = -(+ve) - (+ve) < 0$ , so $f(x)$ is a decreasing {function}
<b>Note:</b>	The final A mark can be scored in part (b) from an incorrect $A = \dots$ or from $A = 0$ or no value of $A$ found in part (a)

## Notes for Question 11 Continued - Alternatives

**(a)**

**Note:** Be aware of the following alternative solutions, by initially dividing by "(x-3)" or "(1-2x)"

$$\bullet \frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{-6x-7}{(1-2x)} - \frac{20}{(x-3)(1-2x)} \equiv 3 - \frac{10}{(1-2x)} - \frac{20}{(x-3)(1-2x)}$$

$$\frac{20}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x) + E(x-3) \Rightarrow D = -4, E = -8$$

$$\Rightarrow 3 - \frac{10}{(1-2x)} - \left( \frac{-4}{(x-3)} + \frac{-8}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2$$

$$\bullet \frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{3x-4}{(x-3)} + \frac{5}{(x-3)(1-2x)} \equiv 3 + \frac{5}{(x-3)} + \frac{5}{(x-3)(1-2x)}$$

$$\frac{5}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x) + E(x-3) \Rightarrow D = -1, E = -2$$

$$\Rightarrow 3 + \frac{5}{(x-3)} + \left( \frac{-1}{(x-3)} + \frac{-2}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2$$

**(b)**

### Alternative Method 1:

$$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)}, x > 3 \Rightarrow f(x) = \frac{1+11x-6x^2}{-2x^2+7x-3}, \left\{ \begin{array}{ll} u=1+11x-6x^2 & v=-2x^2+7x-3 \\ u'=11-12x & v'=-4x+7 \end{array} \right\}$$

$$f'(x) = \frac{(-2x^2+7x-3)(11-12x) - (1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$$

Uses quotient rule  
to find  $f'(x)$

**M1**

Correct differentiation

**A1**

$$f'(x) = \frac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2} \text{ and a correct explanation,}$$

e.g.  $f'(x) = -\frac{(+ve)}{(+ve)} < 0$ , so  $f(x)$  is a decreasing {function}

**A1**

### Alternative Method 2:

Allow M1A1A1 for the following solution:

$$\text{Given } f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)} = 3 + \frac{4}{(x-3)} + \frac{2}{(2x-1)}$$

as  $\frac{4}{(x-3)}$  decreases when  $x > 3$  and  $\frac{2}{(2x-1)}$  decreases when  $x > 3$

then  $f(x)$  is a decreasing {function}