Question	Scheme	Marks	AOs
13	$C: y = x\ln x;$ l is a normal to C at $P(\mathrm{e},\, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\dfrac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\dfrac{1}{2} \Rightarrow y - \mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e}) \Rightarrow x = ...$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$ **or** $\dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\displaystyle\int x\ln x\,\mathrm{d}x = \right\}\ \dfrac{1}{2}x^2\ln x - \displaystyle\int \dfrac{1}{x}.\left(\dfrac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \dfrac{1}{2}x^2\ln x - \displaystyle\int \dfrac{1}{2}x\,\{\mathrm{d}x\}\right\} = \dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...\,;\ \text{Area}(R_2) = \dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \tfrac{1}{4}\mathrm{e}^2 + \tfrac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \tfrac{5}{4}\mathrm{e}^2 + \tfrac{1}{4}$	A1	1.1b
		(10)	

Notes for Question 13

M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } \mathrm{g}'(x))$, where $\mathrm{g}(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e},\, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = ...$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$, with limits of e and 1 and some attempt to substitute these and subtract • **or** Area under line $= \dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \displaystyle\int B\left(\dfrac{x^2}{x}\right)\{\mathrm{d}x\};\ A \neq 0,\ B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2;\ \lambda \neq 0$
A1:	$\dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\tfrac{5}{4}\mathrm{e}^2 + \tfrac{1}{4}$

Notes for Question 13 Continued	
Note:	$\text{Area}(R_2)$ can also be found by integrating the line *l* between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\text{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\text{e}\right) \text{d}x = [\ \ldots\ ]_{\text{e}}^{\text{their } x_A} = \ldots$
Note:	**<u>Calculator approach with no algebra, differentiation or integration seen:</u>** • Finding *l* cuts through the *x*-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the *x*-axis between $x = 1$ and $x = \text{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972... + 7.3890... = 9.4862...$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.