Question	Scheme	Marks	AOs
14	$N=\dfrac{900}{3+7e^{-0.25t}}=900(3+7e^{-0.25t})^{-1}, t\in\mathbb{R}, t\geq 0;\ \dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow\dfrac{dN}{dt}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200}=\dfrac{\left(\dfrac{900}{3+7e^{-0.25t}}\right)\left(300-\dfrac{900}{3+7e^{-0.25t}}\right)}{1200}$	dM1	2.1
	LHS $=\dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e., RHS $=\dfrac{900(300(3+7e^{-0.25t})-900)}{1200(3+7e^{-0.25t})^2}=\dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. and states hence $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N=150$ (can be implied)	B1	2.2a
	so $150=\dfrac{900}{3+7e^{-0.25T}}\Rightarrow e^{-0.25T}=\dfrac{3}{7}$	M1	3.4
	$T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Notes for Question 14	
14 (b)	
M1:	Attempts to differentiate using • the chain rule to give $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \pm A\mathrm{e}^{-0.25t}(3+7\mathrm{e}^{-0.25t})^{-2}$ or $\dfrac{\pm A\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e. • the quotient rule to give $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{(3+7\mathrm{e}^{-0.25t})(0) \pm A\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ • implicit differentiation to give $N(3+7\mathrm{e}^{-0.25t}) = 900 \Rightarrow (3+7\mathrm{e}^{-0.25t})\dfrac{\mathrm{d}N}{\mathrm{d}t} \pm AN\mathrm{e}^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7\mathrm{e}^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7\mathrm{e}^{-0.25t})\dfrac{\mathrm{d}N}{\mathrm{d}t} - 1.75N\mathrm{e}^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating t, to form an equation linking $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ and N only **Way 2:** Complete substitution of $N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}}$ into $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7\mathrm{e}^{-0.25t} = \dfrac{900}{N}$ and $\mathrm{e}^{-0.25t} = \dfrac{900}{N}$ or substitutes $\mathrm{e}^{-0.25t} = \dfrac{\frac{900}{N}-3}{7}$ into their $\dfrac{\mathrm{d}N}{\mathrm{d}t} = ...$ to form an equation linking $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ and N
A1*:	**Way 1:** Correct algebra leading to $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ is maximised when $N = 150$
M1:	Uses the model $N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}}$ with their $N = 150$ and proceeds as far as $\mathrm{e}^{-0.25T} = k,\ k > 0$ or $\mathrm{e}^{0.25T} = k,\ k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\dfrac{\mathrm{d}^2N}{\mathrm{d}t^2} = \dfrac{\mathrm{d}N}{\mathrm{d}t}\left(\dfrac{300}{1200} - \dfrac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \dfrac{900}{3+7\mathrm{e}^{-0.25t}} \Rightarrow t = ...$ or $0 = \dfrac{900}{3+7\mathrm{e}^{-0.25t}} \Rightarrow t = ...$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
14	$N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}} = 900(3+7\mathrm{e}^{-0.25t})^{-1},\ t \in \mathbb{R},\ t \geq 0\ ;\ \ \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}$		
(b) Way 3	$\displaystyle\int \frac{1}{N(300-N)}\,\mathrm{d}N = \int \frac{1}{1200}\,\mathrm{d}t$	M1	2.1
	$\displaystyle\int \frac{1}{300}\left(\frac{1}{N} + \frac{1}{300-N}\right)\mathrm{d}N = \int \frac{1}{1200}\,\mathrm{d}t$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t\ \{+c\}$	A1	1.1b
	$\{t=0,\ N=90 \Rightarrow\}\ \ c = \dfrac{1}{300}\ln(90) - \dfrac{1}{300}\ln(210) \Rightarrow c = \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t + \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\ln N - \ln(300-N) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right)$ $\ln\left(\dfrac{N}{300-N}\right) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right)\ \Rightarrow\ \dfrac{N}{300-N} = \dfrac{3}{7}\mathrm{e}^{\frac{1}{4}t}$	dM1	2.1
	$7N = 3\mathrm{e}^{\frac{1}{4}t}(300-N) \Rightarrow 7N + 3N\mathrm{e}^{\frac{1}{4}t} = 900\mathrm{e}^{\frac{1}{4}t}$ $N(7+3\mathrm{e}^{\frac{1}{4}t}) = 900\mathrm{e}^{\frac{1}{4}t} \Rightarrow N = \dfrac{900\mathrm{e}^{\frac{1}{4}t}}{7+3\mathrm{e}^{\frac{1}{4}t}} \Rightarrow N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}}$ *	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7\mathrm{e}^{-0.25t}) = 900 \Rightarrow \mathrm{e}^{-0.25t} = \dfrac{1}{7}\left(\dfrac{900}{N} - 3\right) \Rightarrow \mathrm{e}^{-0.25t} = \dfrac{900-3N}{7N}$	M1	2.1
	$\Rightarrow t = -4\big(\ln(900-3N) - \ln(7N)\big)$ $\Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N} = -4\left(\dfrac{-3}{900-3N} - \dfrac{7}{7N}\right)$	A1	1.1b
	$\dfrac{\mathrm{d}t}{\mathrm{d}N} = 4\left(\dfrac{1}{300-N} + \dfrac{1}{N}\right) \Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N} = 4\left(\dfrac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\dfrac{\mathrm{d}t}{\mathrm{d}N} = \left(\dfrac{1200}{N(300-N)}\right) \Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	

Notes for Question 14 Continued	
(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give $\ln$ terms $= kt\ \{+c\},\ k \neq 0$, with or without a constant of integration c
A1:	$\frac{1}{300}\ln N - \frac{1}{300}\ln(300-N) = \frac{1}{1200}t\ \{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t=0,\ N=90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t} = f(N);\ \lambda \neq 0$ or $\lambda e^{-\frac{1}{4}t} = f(N);\ \lambda \neq 0$
A1*:	Correct manipulation leading to $N = \frac{900}{3+7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\frac{dt}{dN} = -4\left(\frac{-3}{900-3N} - \frac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ *