

Question	Scheme		Marks	AOs
9 (a)	$\{y = ab^t \Rightarrow \log_{10} y = \log_{10}(ab^t) \Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^t$		M1	1.1b
	$\Rightarrow \log_{10} y = t \log_{10} b + \log_{10} a, \{ \text{where } c = \log_{10} a \}$		A1	2.1
			(2)	
(b)	$c = \log_{10} a = 2.23,$	$m = \log_{10} b = -0.076$		
	$\Rightarrow a = 10^{2.23} \{= 169.8243652...\}$	$\Rightarrow b = 10^{-0.076} \{= 0.8394599865...\}$	M1	1.1b
	$a = 170 \text{ (2 sf) and } b = 0.84 \text{ (2 sf)}$		A1	1.1b
			(2)	
(c)	$y = (170)(0.84)^t$			
	(i) $\{a = "170" \Rightarrow \}$ e.g. “170” milligrams of antibiotic were initially given to the patient - the initial dose of the antibiotic is estimated as “170” milligrams		B1ft	3.4
	(ii) $\{b = "0.84" \Rightarrow \}$ e.g. after the antibiotic is first given the amount of antibiotic in the patient’s bloodstream reduces by approximately “16%” per hour		B1ft	3.4
			(2)	
(d)	$30 = (170)(0.84)^t \Rightarrow \frac{30}{170} = (0.84)^t \Rightarrow \ln\left(\frac{30}{170}\right) = \ln(0.84)^t$ $\Rightarrow \ln\left(\frac{30}{170}\right) = t \ln(0.84) \Rightarrow t = \frac{\ln\left(\frac{30}{170}\right)}{\ln(0.84)}$		M1	3.4
	$\{t = 9.948766031... \Rightarrow \} \quad t = 9.9(\text{hours}) \text{ (1 dp)}$		A1	1.1b
			(2)	
(d) Alt 1	$\log_{10} 30 = -0.076t + 2.23 \Rightarrow t = \frac{2.23 - \log_{10} 30}{0.076}$		M1	3.4
	$\{t = 9.90629928... \Rightarrow \} \quad t = 9.9 \text{ (hours) (1 dp)}$		A1	1.1b
			(2)	
(e)	e.g. As $t = 9.9$ is outside of the experimental data $0 \leq t \leq 5$, we do not have enough evidence to deduce that the model $y = (170)(0.84)^t$ is still valid. So, the estimate in part (d) should be treated with caution.		B1	3.5b
			(1)	
(9 marks)				

Question 9 Notes:

(a)

M1: Starting from $y = ab^t$, takes logs of both sides and uses the addition law of logarithms to progress as far as $\log_{10} y = \log_{10} a + \log_{10} b^t$

A1: Starting from $y = ab^t$, correctly shows that $\log_{10} y = t \log_{10} b + \log_{10} a$ with no errors seen

Note: M1 (special case) can be given in part (a) for stating $c = \log_{10} a$

(b)

M1: For either $a = 10^{2.23}$ or $b = 10^{-0.076}$

A1: $a = 170$ and $b = 0.84$

(c)(i)

B1ft: Correct practical interpretation of their a , where their $a > 0$

(c)(ii)

B1ft: Correct practical interpretation of their b , where their $b: 0 < b < 1$

(d)

M1: Substitutes $y = 2.5$ into the model $y = (\text{their } a)(\text{their } b)^t$ and rearranges their equation to give $t = \dots$

A1: 9.9 (hours) (1 dp)

(d)

Alt 1

M1: Substitutes $y = 30$, $m = -0.076$ and $c = 2.23$ into the model $\log_{10} y = mt + c$ and rearranges their equation to give $t = \dots$

A1: 9.9 (hours) (1 dp)

(e)

B1: E.g. Estimate should be treated with caution because $t = 9.9$ is outside the range of times, i.e. $0 \leq t \leq 5$, for which the model $y = (170)(0.84)^t$ is valid