

Question	Scheme	Marks	AOs
<b>10 (a)</b>	$\frac{dA}{dt} \propto \sqrt{A} \Rightarrow \frac{dA}{dt} = k\sqrt{A}$ or $\frac{dA}{dt} = kA^{\frac{1}{2}}$	B1	3.1b
	$\int \frac{1}{A^{\frac{1}{2}}} dA = \int k dt$	M1	1.1b
	$\left\{ \int A^{-\frac{1}{2}} dA = \int k dt \Rightarrow \right\} \frac{A^{\frac{1}{2}}}{(\frac{1}{2})} = kt \{+c\}$ or $2A^{\frac{1}{2}} = kt \{+c\}$	A1	1.1b
	$\{t=0, A=9 \Rightarrow\} 2\sqrt{9} = k(0) + c$	M1	3.4
	$\Rightarrow c = 6 \Rightarrow 2A^{\frac{1}{2}} = kt + 6$ $\{t=6, A=56.25 \Rightarrow\} 2\sqrt{56.25} = k(6) + 6$	dM1	1.1b
	$\Rightarrow 15 = 6k + 6 \Rightarrow k = \frac{9}{6} \Rightarrow k = \frac{3}{2}$ $\Rightarrow 2A^{\frac{1}{2}} = \frac{3}{2}t + 6 \Rightarrow A^{\frac{1}{2}} = \frac{3}{4}t + 3 \Rightarrow A = \left(\frac{3}{4}t + 3\right)^2 *$	A1*	2.1
		<b>(6)</b>	
<b>(b)</b> <b>(i), (ii)</b>	<p><b>Either</b></p> <ul style="list-style-type: none"> <li><math>t = 12, A = \left(\frac{3}{4}(12) + 3\right)^2 = 144 \{\approx 143.78\}</math></li> <li><math>t = 18, A = 272.25 \{\approx 271.19\}</math></li> <li><math>t = 24, A = 441 \{&gt; 334.81\}</math></li> <li><math>\{t = 30, A = 650.25 \{&gt; 337.33\}\}</math></li> </ul> <p><b>or</b></p> <ul style="list-style-type: none"> <li><math>A = 143.78 \Rightarrow 143.78 = \left(\frac{3}{4}t + 3\right)^2 \Rightarrow t = 11.98777... \{\approx 12\}</math></li> <li><math>A = 271.19 \Rightarrow t = 17.95713... \{\approx 18\}</math></li> <li><math>A = 334.81 \Rightarrow t = 20.39709... \{&lt; 24\}</math></li> <li><math>\{A = 337.33 \Rightarrow t = 20.48873... \{&lt; 30\}\}</math></li> </ul>	M1	3.4
	Biologist's model works well for $t = 12$ and $t = 18$ but appears to give an overestimate for $A$ (or does not work well) when $t = 24$ and $t = 30$	A1	3.5a
	<p>E.g.</p> <ul style="list-style-type: none"> <li>The biologist's model appears to break down for large values of <math>t</math>. This may be because the biologist's model predicts values for <math>A</math> which are greater than the total surface area of the piece of bread used in the experiment.</li> <li>The biologist's results indicate an upper limit for <math>A</math>, but the biologist's model does not give an upper limit for <math>A</math>.</li> </ul>	B1	3.2a
		<b>(3)</b>	

**(9 marks)**

## Question 10 Notes:

(a)

**B1:** Translates the biologist's model regarding proportionality into a differential equation, which involves a constant of proportionality. E.g.  $\frac{dA}{dt} \propto \sqrt{A} \Rightarrow \frac{dA}{dt} = k\sqrt{A}$

**M1:** Correct method of separating the variables  $A$  and  $t$  in their differential equation

**A1:**  $\frac{A^{\frac{1}{2}}}{(\frac{1}{2})} = kt$  or  $2A^{\frac{1}{2}} = kt$ , with or without a constant of integration

**M1:** Some evidence of applying the measurements  $t = 0, A = 9$  or  $A = 9.00$  to a changed equation containing a constant of integration. e.g.  $c$

**dM1:** **dependent on the previous M mark**

Applies  $t = 6, A = 56.25$  and their value of  $c$  to their changed equation which contains their constant of proportionality

**A1\*:** Shows that  $A = \left(\frac{3}{4}t + 3\right)^2$ , with no errors in their working

(b)

(i), (ii)

**M1:** Uses the model found in part (a) to find

- **either** values for  $A$  when  $t = 12, t = 18$  and  $t = 24$
- **or** values for  $t$  when  $A = 143.78, A = 271.19$  and  $A = 334.81$

**A1:** • **Either**  $t = 12 \Rightarrow A = 144, t = 18 \Rightarrow A = \text{awrt } 272$  **and**  $t = 24 \Rightarrow A = 441$

- **or**  $A = 143.78 \Rightarrow t = \text{awrt } 12, A = 271.19 \Rightarrow t = \text{awrt } 18$  **and**  $A = 334.81 \Rightarrow t = \text{awrt } 20$
- and** evaluates (see scheme) the outcomes of the model

**B1:** See scheme