Question	Scheme	Marks	AOs
15	$a_{n+1} = k - \dfrac{3k}{a_n},\ \ n \in \mathbb{Z}^+;\ k$ is a constant Sequence $a_1, a_2, a_3, \ldots$ where $a_2 = 2$ is periodic of order 3		
(a)	$a_3 = k - \dfrac{3k}{2} = -\dfrac{1}{2}k\,;\qquad a_4 = k - \dfrac{3k}{\left(-\frac{1}{2}k\right)} = k+6$	M1	1.1b
	$\{a_5 = a_2 \Rightarrow\}\ a_5 = k - \dfrac{3k}{k+6} = 2$	M1	3.1a
	$\Rightarrow k(k+6) - 3k = 2(k+6) \Rightarrow k^2 + 6k - 3k = 2k + 12$ $\Rightarrow k^2 + k - 12 = 0\ *$	A1*	2.1
		(3)	
(b)	$(k+4)(k-3) = 0 \Rightarrow k = -4, 3$	M1	3.1a
	$k = 3;\ \ \{a_2 = 2,\}\ a_3 = -\dfrac{3}{2},\ a_4 = 9$ $\{\,k = -4;\ \{a_2 = 2,\}\ a_3 = 2\ \{\Rightarrow a_4 = 2,\ a_1 = 2\,;$ so reject as $a_1 = a_2\}\,\}$	A1	1.1b
	Note: $k = 3;\ \ a_1 = 9, a_2 = 2,\ a_3 = -\dfrac{3}{2},\ a_4 = 9$, etc.		
	$\displaystyle\sum_{r=1}^{121} a_r = 40\left(2 - \frac{3}{2} + 9\right) + 9$	M1	2.2a
	$= 40(9.5) + 9 = 380 + 9 = 389$	A1	1.1b
		(4)	
			(7 marks)

Question 15 Notes:

(a)	
M1:	Uses $a_2 = 2$ to find **both** a_3 in terms of k (which can be un-simplified or simplified) **and** a_4 in terms of k (which can be un-simplified or simplified)
M1:	Shows understanding that the sequence is periodic of order 3 by applying complete strategy of finding a_5 in terms of k and setting the result equal to 2 (which is the same as a_2)
A1*:	Shows that $k^2 + k - 12 = 0$ with no errors in their working
(b)	
M1:	Complete process of finding and using $k = 3$ to find ***the values*** of either a_3 and a_4 **or** a_1 and a_3
A1:	Uses $k = 3$ to find $a_3 = -\dfrac{3}{2}$ and $a_4 = 9$ **or** $a_1 = 9$ and $a_3 = -\dfrac{3}{2}$
M1:	Deduces $\displaystyle\sum_{r=1}^{121} a_r = 40\left(2 + \text{"}-1.5\text{"} + \text{"}9\text{"}\right) + \text{"}9\text{"}$
A1:	389