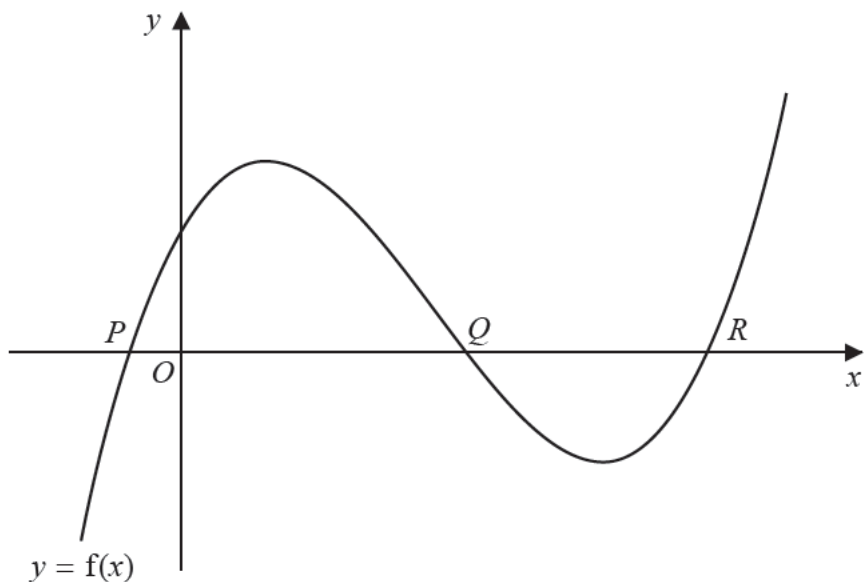**4.**



**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x^3 - 6x^2 + 7x + 2 \qquad x \in \mathbb{R}$$

The curve cuts the $x$-axis at the points $P$, $Q$ and $R$, as shown in Figure 1.

The coordinates of $Q$ are $(2, 0)$

(a) Write $f(x)$ as a product of two algebraic factors.

**(2)**

(b) Find, giving your answer in simplest form,

   (i) the exact $x$ coordinate of $P$,

   (ii) the exact $x$ coordinate of $R$.

**(2)**

(c) Deduce the number of real solutions, for $-\pi \leqslant \theta \leqslant 12\pi$, to the equation

$$\sin^3 \theta - 6\sin^2 \theta + 7\sin\theta + 2 = 0$$

   justifying your answer.

**(2)**