7.

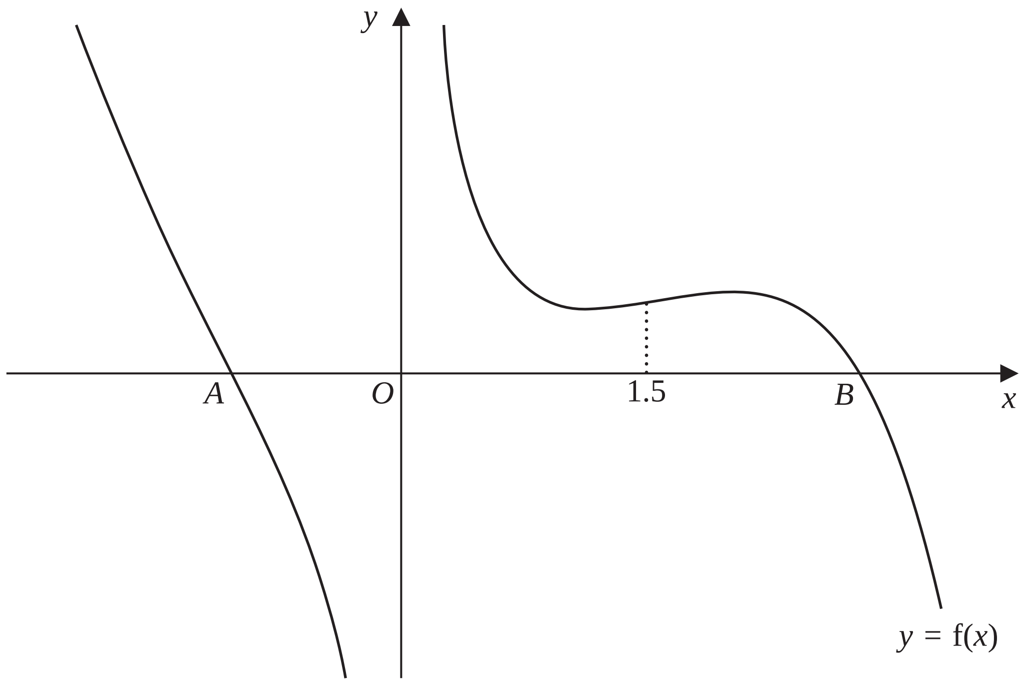


Figure 3

Figure 3 shows a plot of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{2}{x} - \mathrm{e}^{x} + 2x^{2} \qquad x \in \mathbb{R}, x \neq 0$$

The curve cuts the x-axis at the point A, where $x = \alpha$, and at the point B, where $x = \beta$, as shown in Figure 3.

(a) Show that α lies between -1.5 and -1

(2)

(b) The iterative formula

$$x_{n+1} = -\sqrt{\left(\frac{1}{2}\mathrm{e}^{x_n} - \frac{1}{x_n}\right)} \qquad n \in \mathbb{N}$$

with $x_1 = -1$ can be used to estimate the value of α.

(i) Find the value of x_3 to 4 decimal places.

(ii) Find the value of α correct to 2 decimal places.

(2)

The value of β lies in the interval [1.5, 3]

A student takes 3 as her first approximation to β.

Given $f(3) = -1.4189$ and $f'(3) = -8.3078$ to 4 decimal places,

(c) apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to β. Give your answer to 2 decimal places.

(2)

A different student takes a starting value of 1.5 as his first approximation to β.

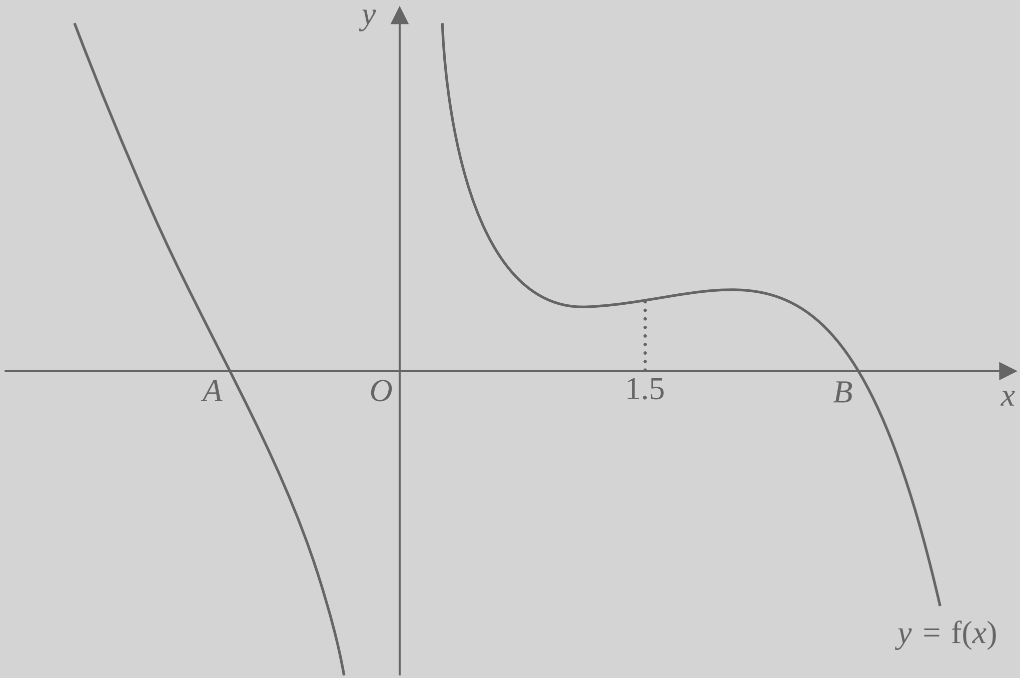
(d) Use Figure 3 to explain whether or not the Newton-Raphson method with this starting value gives a good second approximation to β.

(2)

[If you need to rework your answer to part (d) turn over for a spare copy of Figure 3]

Question 7 continued

Only use this spare copy of Figure 3 if you have to rework your answer to part (d).



Spare copy of Figure 3