

Question	Scheme	Marks	AOs
5	States $\left\{ \lim_{\delta x \rightarrow 0} \sum_{x=4}^9 \sqrt{x} \delta x \text{ is} \right\} \int_4^9 \sqrt{x} dx$	B1	1.2
	$= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_4^9$	M1	1.1b
	$= \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3} \text{ or } 12\frac{2}{3} \text{ or awrt } 12.7$	A1	1.1b
		(3)	

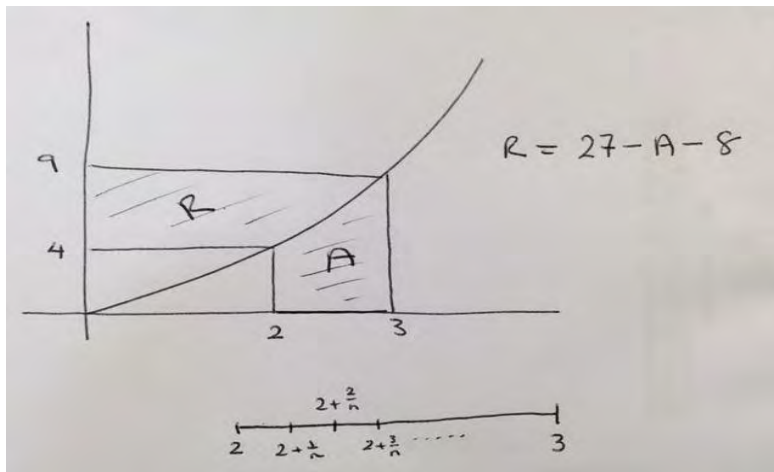
(3 marks)

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x} dx$ with or without the 'dx'
M1:	Integrates \sqrt{x} to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}} \right]_4^9$ or for $\lambda \times 9^{\frac{3}{2}} - \lambda \times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x} dx - \int_1^3 \sqrt{x} dx$ or for $\int_3^9 \sqrt{x} dx$ without reference to a correct $\int_4^9 \sqrt{x} dx$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x} dx = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3} x^{\frac{3}{2}} + c \right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x} dx$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Alt

The following method is correct:



$$\begin{aligned}
 \text{Area (A)} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (x_i - x_{i-1})f(x_i) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(2 + \frac{i}{n} \right)^2 \\
 &= \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{i=1}^n 4 + \frac{1}{n} \sum_{i=1}^n \left(\frac{4i}{n} \right) + \frac{1}{n} \sum_{i=1}^n \left(\frac{i^2}{n^2} \right) \right] \\
 &= \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{i=1}^n 4 + \frac{4}{n^2} \sum_{i=1}^n i + \frac{1}{n^3} \sum_{i=1}^n i^2 \right] \\
 &= \lim_{n \rightarrow \infty} \left[\frac{4n}{n} + \frac{4}{n^2} \left(\frac{1}{2} n(n+1) \right) + \frac{1}{n^3} \left(\frac{1}{6} n(n+1)(2n+1) \right) \right] \\
 &= \lim_{n \rightarrow \infty} \left[\frac{4}{n} + \frac{4n^2 + 4n}{2n^2} + \frac{2n^3 + 3n^2 + n}{6n^3} \right] \\
 &= \lim_{n \rightarrow \infty} \left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} \right] \\
 &= 4 + 2 + \frac{1}{3} = \frac{19}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{So, } \lim_{\delta x \rightarrow 0} \sum_{x=4}^9 \sqrt{x} \delta x &= \text{Area}(R) = (3 \times 9) - (2 \times 4) - \frac{19}{3} \\
 &= \frac{38}{3} \quad \text{or } 12\frac{2}{3} \quad \text{or awrt 12.7}
 \end{aligned}$$