$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Question	Scheme	Marks	AOs
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	14 (a)	$\{u=4-\sqrt{h} \Rightarrow\} \frac{\mathrm{d}u}{\mathrm{d}h} = -\frac{1}{2}h^{-\frac{1}{2}} \text{ or } \frac{\mathrm{d}h}{\mathrm{d}u} = -2(4-u) \text{ or } \frac{\mathrm{d}h}{\mathrm{d}u} = -2\sqrt{h}$	B1	1.1b
		$\left\{\int \frac{\mathrm{d}h}{4-\sqrt{h}} = \right\} \int \frac{-2(4-u)}{u} \mathrm{d}u$	M1	2.1
		$=\int \left(-\frac{8}{u}+2\right) \mathrm{d}u$	M1	1.1b
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$-\frac{8\ln y+2y(1-z)}{2}$	M1	1.1b
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$=-6 \lim u + 2u \{\pm c\}$	A1	1.1b
$ \begin{array}{c} \textbf{(b)} & \left\{ \frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow \right\} 4-\sqrt{h} = 0 & \text{M1} & 3.4 \\ \hline \text{Deduces any of } 0 < h < 16, \ 0 \leq h < 16, \ 0 < h \leq 16, \ 0 \leq h \leq 16, \ 0 \leq h \leq 16, \\ h < 16, \ h \leq 16 \ \text{ or all values up to 16} & \text{A1} & 2.2a \\ \hline \textbf{(c)} & \textbf{(2)} & \hline \textbf{(2)} $		$= -8\ln 4 - \sqrt{h} + 2(4 - \sqrt{h}) + c = -8\ln 4 - \sqrt{h} - 2\sqrt{h} + k *$	A1*	2.1
$\frac{(u - 20)^{-1}}{(1 - \sqrt{h})^{-1}} = \frac{(u - 10)^{-1}}{(1 - \sqrt{h})^$			(6)	
$\frac{h < 16, h \le 16 \text{ or all values up to } 16}{(2)}$ $\frac{h < 16, h \le 16 \text{ or all values up to } 16}{(2)}$ $\frac{h < 16, h \le 16 \text{ or all values up to } 16}{(2)}$ $\frac{h < 16, h \le 16 \text{ or all values up to } 16}{(2)}$ $\frac{(2)}{Way 1}$ $\frac{\int \frac{1}{(4 - \sqrt{h})} dh = \int \frac{1}{20} t^{0.25} dt}{125} \frac{H}{4} + \frac{11.1b}{A1}$ $\frac{1.1b}{A1}$	(b)	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20} = 0 \implies \right\} 4-\sqrt{h} = 0$	M1	3.4
$\begin{array}{c c} (c) \\ \textbf{Way 1} & \int \frac{1}{(4-\sqrt{h})} dh = \int \frac{1}{20} t^{0.25} dt \\ & B1 & 1.1b \\ \hline & & \\$		•	A1	2.2a
$\frac{-8\ln 4 - \sqrt{h} - 2\sqrt{h} = \frac{1}{25}t^{1.25} + c}{1.25} + c} \qquad \frac{M1}{A1} \qquad \frac{1.1b}{A1}$ $\frac{-8\ln 4 - \sqrt{h} - 2\sqrt{h} = \frac{1}{25}t^{1.25} + c}{1.25} \qquad M1 \qquad 3.4$ $\Rightarrow c = -8\ln(3) - 2 \Rightarrow -8\ln 4 - \sqrt{h} - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\frac{h = 12 \Rightarrow}{2} - 8\ln 4 - \sqrt{12} - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\frac{h = 12 \Rightarrow}{2} - 8\ln 4 - \sqrt{12} - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\frac{t^{1.25} = 221.2795202 \Rightarrow t = \frac{1.25}{221.2795} \text{ or } t = (221.2795)^{0.8} \qquad M1 \qquad 1.1b$ $\frac{t = 75.154 \Rightarrow t = 75.2 (years) (3 sf) \text{ or awrt } 75.2 (years) \qquad A1 \qquad 1.1b$ $\frac{(c)}{Way 2} \qquad \int_{1}^{12} \frac{20}{(4 - \sqrt{h})} dh = \int_{0}^{T} t^{0.25} dt \qquad B1 \qquad 1.1b$ $\frac{1.1b}{A1} \qquad 1.1b$ $\frac{20(-8\ln 4 - \sqrt{h} - 2\sqrt{h})}{1} \Big]_{1}^{12} = \left[\frac{4}{5}t^{1.25}\right]_{0}^{T} \qquad \frac{M1}{A1} \qquad 1.1b$ $\frac{1.1b}{A1} \qquad 1.1b$ $1.$			(2)	
$\frac{-8\ln 4-\sqrt{h} -2\sqrt{h}=\frac{1}{25}t^{1.25} \{+c\}}{A1}$ A1 1.1b $\frac{1}{25}t^{1.25} \{+c\}$ A1 1.1b $\frac{1}{25}t^{1.25} + c$ M1 3.4 $\Rightarrow c = -8\ln(3) - 2 \Rightarrow -8\ln 4-\sqrt{h} - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\frac{1}{25}t^{1.25} - 8\ln(3) - 2$ M1 1.1b $\frac{1}{25}t^{1.25} = 221.2795202 \Rightarrow t = \frac{1.25}{221.2795} \text{ or } t = (221.2795)^{0.8}$ M1 1.1b $\frac{1}{20}(-8\ln 4-\sqrt{h} - 2\sqrt{h}) \int_{1}^{12} = \left[\frac{4}{5}t^{1.25}\right]_{0}^{7}$ M1 1.1b $\frac{1}{20}(-8\ln(4-\sqrt{h} - 2\sqrt{h}) \int_{1}^{12} = \left[\frac{4}{5}t^{1.25}\right]_{0}^{7}$ M1 1.1b $\frac{1}{3.1a}$ $\frac{1}{1.1b}$ $\frac{1}{20}(-8\ln(4-\sqrt{h} - 2\sqrt{h}) \int_{1}^{12} = \left[\frac{4}{5}t^{1.25}\right]_{0}^{7}$ M1 1.1b $\frac{1}{3.1a}$ $\frac{1}{1.1b}$ A1 1.1b $\frac{1}{1.1b}$ A1 1.1b A1 A1 1.1b A1 A1		$\int \frac{1}{(4 - \sqrt{h})} \mathrm{d}h = \int \frac{1}{20} t^{0.25} \mathrm{d}t$	B1	1.1b
$\frac{\{t=0, h=1 \Rightarrow\} -8\ln(4-1) - 2\sqrt{(1)} = \frac{1}{25}(0)^{1.25} + c}{(125 - 8\ln(3) - 2)} \qquad M1 \qquad 3.4$ $\Rightarrow c = -8\ln(3) - 2 \Rightarrow -8\ln 4 - \sqrt{h} - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\frac{\{h=12 \Rightarrow\} -8\ln 4 - \sqrt{12} - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2}{t^{1.25} - 221.2795202 \Rightarrow t = \frac{1.25}{221.2795} \text{ or } t = (221.2795)^{0.8} \qquad M1 \qquad 1.1b$ $t = 75.154 \Rightarrow t = 75.2 \text{ (years) } (3 \text{ sf) or awrt } 75.2 \text{ (years)} \qquad A1 \qquad 1.1b$ $\frac{(c)}{Way 2} \qquad \int_{1}^{12} \frac{20}{(4 - \sqrt{h})} dh = \int_{0}^{T} t^{0.25} dt \qquad B1 \qquad 1.1b$ $\frac{1.1b}{A1} \qquad 1.1b$ $\frac{20(-8\ln 4 - \sqrt{h} - 2\sqrt{h})}{1^{12}} = \left[\frac{4}{5}t^{1.25}\right]_{0}^{T} \qquad M1 \qquad 3.4a$ $\frac{1.1b}{A1} \qquad 1.1b$ $\frac{20(-8\ln(4 - \sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4 - 1) - 2\sqrt{1})}{1^{12}} = \left[\frac{4}{5}t^{1.25} - 0\right]_{0}^{T} \qquad M1 \qquad 3.4a$ $\frac{1.1b}{A1} \qquad 1.1b$		$81n 4$ \sqrt{h} $2\sqrt{h} = \frac{1}{t^{1.25}}(1, a)$	M1	1.1b
$\frac{2.5}{(h = 12 \Rightarrow)} = -8\ln(3) - 2 \Rightarrow -8\ln 4 - \sqrt{h} - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\frac{h = 12 \Rightarrow}{h = 12 \Rightarrow} -8\ln 4 - \sqrt{12} - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\frac{t^{1.25} = 221.2795202 \Rightarrow t = \frac{1.25}{221.2795} \text{ or } t = (221.2795)^{0.8} \qquad \text{M1} \qquad 1.1\text{ b}$ $\frac{t = 75.154 \Rightarrow t = 75.2 \text{ (years)} (3 \text{ sf) or awrt 75.2 (years)} \qquad \text{A1} \qquad 1.1\text{ b}$ $\frac{(c)}{\text{Way 2}} \qquad \int_{1}^{12} \frac{20}{(4 - \sqrt{h})} dh = \int_{0}^{T} t^{0.25} dt \qquad \text{B1} \qquad 1.1\text{ b}$ $\frac{1.19}{41} \qquad 1.16 $		$\frac{-8m 4-\sqrt{n} -2\sqrt{n}-\frac{1}{25}t}{5}$	A1	1.1b
$\frac{\{h = 12 \Rightarrow\} -8\ln 4 - \sqrt{12} - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2}{t^{1.25} - 221.2795202 \Rightarrow t = \frac{1.25}{221.2795} \text{ or } t = (221.2795)^{0.8}}{\text{M1}} \frac{M1}{1.1b}$ $\frac{t^{1.25} = 221.2795202 \Rightarrow t = 75.2 \text{ (years)} (3 \text{ sf) or awrt } 75.2 \text{ (years)}}{1.1b} \frac{M1}{1.1b}$ $\frac{1.1b}{\text{Note: You can recover work for part (c) in part (b)}}{\int_{1}^{12} \frac{20}{(4 - \sqrt{h})} dh} = \int_{0}^{T} t^{0.25} dt} \frac{M1}{1.1b}$ $\frac{1.1b}{1.1b}$ $\frac{20(-8\ln 4 - \sqrt{h} - 2\sqrt{h})}{1} = \left[\frac{4}{5}t^{1.25}\right]_{0}^{T}} \frac{M1}{1.1b}$ $\frac{1.1b}{1.1b}$ $\frac{20(-8\ln(4 - \sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4 - 1) - 2\sqrt{1})}{1} = \frac{4}{5}T^{1.25} - 0} \frac{M1}{1.1b}$ $\frac{1.1b}{1.1b}$ $\frac{T^{1.23} = 221.2795202 \Rightarrow T = \frac{1.25}{221.2795} \text{ or } T = (221.2795)^{0.8}}{1.1b}$ $\frac{M1}{1.1b}$		$\{t=0, h=1 \Longrightarrow\} -8\ln(4-1) - 2\sqrt{(1)} = \frac{1}{25}(0)^{1.25} + c$	M1	3.4
$\frac{t = 75.154 \Rightarrow t = 75.2 \text{ (years) } (3 \text{ sf) or awrt } 75.2 \text{ (years)}}{\text{Note: You can recover work for part (c) in part (b)}} $ (7) (c) (c) (d) (f) (f) (f) (f) (f) (f) (f) (f) (f) (f			dM1	3.1a
Note: You can recover work for part (c) in part (b)(7)(c) Way 2 $\int_{1}^{12} \frac{20}{(4-\sqrt{h})} dh = \int_{0}^{T} t^{0.25} dt$ B11.1b $\left[20(-8\ln 4-\sqrt{h} -2\sqrt{h}) \right]_{1}^{12} = \left[\frac{4}{5} t^{1.25} \right]_{0}^{T}$ M11.1b $20(-8\ln(4-\sqrt{12})-2\sqrt{12})-20(-8\ln(4-1)-2\sqrt{1}) = \frac{4}{5}T^{1.25} - 0$ M13.4 $T^{1.25} = 221.2795202 \Rightarrow T = \frac{1.25}{221.2795}$ or $T = (221.2795)^{0.8}$ M11.1b $T = 75.154 \Rightarrow T = 75.2$ (years) (3 sf) or awrt 75.2 (years)A11.1b			M1	1.1b
$\begin{array}{c} \textbf{(c)}\\ \textbf{Way 2} & \int_{1}^{12} \frac{20}{(4-\sqrt{h})} dh = \int_{0}^{T} t^{0.25} dt & \text{B1} & 1.1b \\ \hline & \left[20(-8\ln\left 4-\sqrt{h}\right -2\sqrt{h}\right) \right]_{1}^{12} = \left[\frac{4}{5}t^{1.25}\right]_{0}^{T} & \frac{\text{M1}}{1.1b} \\ \hline & A1 & 1.1b \\ \hline & 20(-8\ln(4-\sqrt{12})-2\sqrt{12})-20(-8\ln(4-1)-2\sqrt{1}) = \frac{4}{5}T^{1.25} - 0 & \frac{\text{M1}}{3.1a} \\ \hline & T^{1.25} = 221.2795202 \Rightarrow T = \frac{1.25}{221.2795} \text{ or } T = (221.2795)^{0.8} & \text{M1} & 1.1b \\ \hline & T = 75.154 \Rightarrow T = 75.2 \text{ (years) (3 sf) or awrt 75.2 (years)} & \text{A1} & 1.1b \end{array}$			A1	1.1b
$\begin{bmatrix} 20(-8\ln 4-\sqrt{h} -2\sqrt{h}) \end{bmatrix}_{1}^{12} = \begin{bmatrix} \frac{4}{5}t^{1.25} \end{bmatrix}_{0}^{T} & \frac{M1}{A1} & \frac{1.1b}{A1} \\ \frac{20(-8\ln(4-\sqrt{12})-2\sqrt{12})-20(-8\ln(4-1)-2\sqrt{1}) = \frac{4}{5}T^{1.25}-0}{dM1} & \frac{M1}{3.1a} \\ \frac{T^{1.25}=221.2795202\Rightarrow T = \frac{1.25}{221.2795} \text{ or } T = (221.2795)^{0.8}}{T = 75.154\Rightarrow T = 75.2 \text{ (years) (3 sf) or awrt 75.2 (years)}} & A1 & 1.1b \\ \end{bmatrix}$			(7)	
$\begin{array}{c c} 20(-8\ln(4-\sqrt{12})-2\sqrt{12})-20(-8\ln(4-1)-2\sqrt{1})=\frac{4}{5}T^{1.25}-0 & \frac{M1}{dM1} & \frac{3.4}{3.1a} \\ \hline T^{1.25}=221.2795202\Rightarrow T=\frac{1.25}{221.2795} \text{ or } T=(221.2795)^{0.8} & M1 & 1.1b \\ \hline T=75.154\Rightarrow T=75.2 \text{ (years) (3 sf) or awrt 75.2 (years)} & A1 & 1.1b \end{array}$		$\int_{1}^{12} \frac{20}{(4-\sqrt{h})} dh = \int_{0}^{T} t^{0.25} dt$	B1	1.1b
$\begin{array}{c c} 20(-8\ln(4-\sqrt{12})-2\sqrt{12})-20(-8\ln(4-1)-2\sqrt{1})=\frac{4}{5}T^{1.25}-0 & \frac{M1}{dM1} & \frac{3.4}{3.1a} \\ \hline T^{1.25}=221.2795202\Rightarrow T=\frac{1.25}{221.2795} \text{ or } T=(221.2795)^{0.8} & M1 & 1.1b \\ \hline T=75.154\Rightarrow T=75.2 \text{ (years) (3 sf) or awrt 75.2 (years)} & A1 & 1.1b \end{array}$		$\begin{bmatrix} 20(81n)4 \sqrt{h} & 2\sqrt{h} \end{bmatrix}^{12} - \begin{bmatrix} 4 \\ t^{1.25} \end{bmatrix}^T$	M1	1.1b
$T^{1.25} = 221.2795202 \Rightarrow T = \frac{1.25}{221.2795} \text{ or } T = (221.2795)^{0.8} \text{ M1} 1.1b$ $T = 75.154 \Rightarrow T = 75.2 \text{ (years) (3 sf) or awrt 75.2 (years)} \text{ A1} 1.1b$		~	A1	1.1b
$T^{1.25} = 221.2795202 \Rightarrow T = \frac{1.25}{221.2795} \text{ or } T = (221.2795)^{0.8} \text{ M1} 1.1b$ $T = 75.154 \Rightarrow T = 75.2 \text{ (years) (3 sf) or awrt 75.2 (years)} \text{ A1} 1.1b$		$20(-8\ln(4-\sqrt{12})-2\sqrt{12})-20(-8\ln(4-1)-2\sqrt{1})-\frac{4}{7}T^{1.25}-0$	M1	3.4
$T = 75.154 \Rightarrow T = 75.2 \text{ (years) } (3 \text{ sf}) \text{ or awrt } 75.2 \text{ (years)}$ A1 1.1b			dM1	3.1a
			M1	1.1b
Note: You can recover work for part (c) in part (b)(7)				1.1b
(15 marks)		Note: You can recover work for part (c) in part (b)		

Notes for Question 14		
(a)		
B1:	See scheme. Allow $du = -\frac{1}{2}h^{-\frac{1}{2}}dh$, $dh = -2(4-u)du$, $dh = -2\sqrt{h}du$ o.e.	
M1:	Complete method for applying $u = 4 - \sqrt{h}$ to $\int \frac{dh}{4 - \sqrt{h}}$ to give an expression of the form	
	$\int \frac{k(4-u)}{u} du \; ; \; k \neq 0$	
Note:	Condone the omission of an integral sign and/or du	
M1:	Proceeds to obtain an integral of the form $\int \left(\frac{A}{u} + B\right) \{du\}; A, B \neq 0$	
M1:	$\int \left(\frac{A}{u} + B\right) \{du\} \rightarrow D \ln u + Eu; A, B, D, E \neq 0; \text{ with or without a constant of integration}$	
A1:	$\int \left(-\frac{8}{u}+2\right) \{du\} \rightarrow -8\ln u + 2u; \text{ with or without a constant of integration}$	
A1*:	dependent on all previous marks	
	Substitutes $u = 4 - \sqrt{h}$ into their integrated result and completes the proof by obtaining the	
	printed result $-8\ln 4-\sqrt{h} -2\sqrt{h}+k$.	
	Condone the use of brackets instead of the modulus sign.	
Note:	They must combine $2(4)$ and their $+c$ correctly to give $+k$	
Note:	Going from $-8\ln 4-\sqrt{h} +2(4-\sqrt{h})+c$ to $-8\ln 4-\sqrt{h} -2\sqrt{h}+k$, with no intermediate	
	working or with no incorrect working is required for the final A1* mark.	
Note:	Allow A1* for correctly reaching $-8\ln \left 4-\sqrt{h}\right -2\sqrt{h}+c+8$ and stating $k=c+8$	
Note:	Allow A1* for correctly reaching $-8\ln \left 4 - \sqrt{h}\right + 2(4 - \sqrt{h}) + k = -8\ln \left 4 - \sqrt{h}\right - 2\sqrt{h} + k$	
	Alternative (integration by parts) method for the 2 nd M, 3 rd M and 1 st A mark	
	$\left\{\int \frac{-2(4-u)}{u} du = \int \frac{2u-8}{u} du\right\} = (2u-8)\ln u - \int 2\ln u du = (2u-8)\ln u - 2(u\ln u - u) \{+c\}$	
2 nd M1:	Proceeds to obtain an integral of the form $(Au + B)\ln u - \int A\ln u \{du\}$; $A, B \neq 0$	
3 rd M1:	Integrates to give $D \ln u + Eu$; $D, E \neq 0$; which can be simplified or un-simplified	
	with or without a constant of integration.	
Note:	Give 3^{rd} M1 for $(2u-8)\ln u - 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$	
1 st A1:	Integrates to give $(2u-8)\ln u - 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e.	
	with or without a constant of integration.	
(b) M1:	Uses the context of the model and has an understanding that the tree keeps growing until	
1711.		
	$\frac{dh}{dt} = 0 \Rightarrow 4 - \sqrt{h} = 0$. Alternatively, they can write $\frac{dh}{dt} > 0 \Rightarrow 4 - \sqrt{h} > 0$	
Note:	Accept $h = 16$ or 16 used in their inequality statement for this mark.	
A1:	See scheme	
Note:	A correct answer can be given M1 A1 from any working.	

	Notes for Question 14		
(c)	Way 1		
B1:	Separates the variables correctly. dh and dt should not be in the wrong positions, although		
	this mark can be implied by later working. Condone absence of integral signs.		
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$		
A1:	Correct integration. E.g. $-8\ln 4-\sqrt{h} - 2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln 4-\sqrt{h} - 2\sqrt{h}) = \frac{4}{5}t^{1.25}$		
	$-8\ln\left 4-\sqrt{h}\right +2(4-\sqrt{h}) = \frac{1}{25}t^{1.25} \text{ or } 20(-8\ln\left 4-\sqrt{h}\right +2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$		
	with or without a constant of integration, e.g. k, c or A		
Note:	There is no requirement for modulus signs.		
M1:	Some evidence of <i>applying</i> both $t = 0$ and $h = 1$ to their model (which can be a changed		
	equation) which contains a constant of integration, e.g. k, c or A		
dM1:	dependent on the previous M mark		
	Complete process of finding their constant of integration, followed by applying $h = 12$ and their		
	constant of integration to their changed equation		
M1:	Rearranges their equation to make $t^{\text{their 1.25}} = \dots$ followed by a correct method to give $t = \dots$; $t > 0$		
Note:	$t^{\text{their 1.25}} = \dots$ can be negative, but their ' $t = \dots$ ' must be positive		
Note:	"their 1.25" cannot be 0 or 1 for this mark		
Note:	Do not give this mark if $t^{\text{their 1.25}} = \dots$ (usually $t^{0.25} = \dots$) is a result of substituting $t = 12$ (or $t = 11$)		
	into the given $\frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. Note: They will usually write $\frac{dh}{dt}$ as either 12 or 11.		
A1:	awrt 75.2		
(c)	Way 2		
B1:	Separates the variables correctly. dh and dt should not be in the wrong positions, although		
	this mark can be implied by later working.		
Note:	Integral signs and limits are not required for this mark.		
M1:	Same as Way 1 (ignore limits)		
A1:	Same as Way 1 (ignore limits)		
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in <i>h</i>) and subtracts		
dM1	dependent on the previous M mark		
	Complete process of applying limits of 1 and 12 and 0 and T (or 't') appropriately to their abarrand equation		
M1.	changed equation		
M1:	Same as Way 1 Same as Way 1		
A1:	Same as way I		