Question	Scheme	Marks	AOs
14 (a)	$\{u = 4-\sqrt{h} \Rightarrow\}\ \dfrac{du}{dh} = -\dfrac{1}{2}h^{-\frac{1}{2}}$ or $\dfrac{dh}{du} = -2(4-u)$ or $\dfrac{dh}{du} = -2\sqrt{h}$	B1	1.1b
	$\left\{\displaystyle\int \frac{dh}{4-\sqrt{h}} = \right\}\ \displaystyle\int \frac{-2(4-u)}{u}\,du$	M1	2.1
	$= \displaystyle\int\left(-\frac{8}{u}+2\right)du$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c\ =\ -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}\ + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\dfrac{dh}{dt} = \dfrac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0<h<16,\ 0\le h<16,\ 0<h\le 16,\ 0\le h\le 16,$ $h<16,\ h\le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\displaystyle\int \frac{1}{(4-\sqrt{h})}\,dh\ =\ \int \frac{1}{20}t^{0.25}\,dt$	B1	1.1b
	$-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}\ = \dfrac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t=0\,,\ h=1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)}\ = \dfrac{1}{25}(0)^{1.25}\ + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2\ \Rightarrow\ -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}\ = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h=12 \Rightarrow\}\ -8\ln\left\|4-\sqrt{12}\right\| - 2\sqrt{12}\ = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202...\ \Rightarrow\ t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t =\ 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
(c) Way 2	$\displaystyle\int_1^{12} \frac{20}{(4-\sqrt{h})}\,dh\ =\ \int_0^T t^{0.25}\,dt$	B1	1.1b
	$\left[20(-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h})\right]_1^{12} = \left[\dfrac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \dfrac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202...\ \Rightarrow\ T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T =\ 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
			(15 marks)

Notes for Question 14	
(a)	
B1:	See scheme. Allow $\mathrm{d}u = -\frac{1}{2}h^{-\frac{1}{2}}\mathrm{d}h$, $\mathrm{d}h = -2(4-u)\mathrm{d}u$, $\mathrm{d}h = -2\sqrt{h}\,\mathrm{d}u$ o.e.
M1: **Note:**	Complete method for applying $u = 4-\sqrt{h}$ to $\int \frac{\mathrm{d}h}{4-\sqrt{h}}$ to give an expression of the form $\int \frac{k(4-u)}{u}\mathrm{d}u$; $k \neq 0$ Condone the omission of an integral sign and/or $\mathrm{d}u$
M1:	Proceeds to obtain an integral of the form $\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\}$; $A, B \neq 0$
M1:	$\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\} \to D\ln u + Eu$; $A, B, D, E \neq 0$; with or without a constant of integration
A1:	$\int\left(-\frac{8}{u}+2\right)\{\mathrm{d}u\} \to -8\ln u + 2u$; with or without a constant of integration
A1*:	**dependent on all previous marks** Substitutes $u = 4-\sqrt{h}$ into their integrated result and completes the proof by obtaining the printed result $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$. Condone the use of brackets instead of the modulus sign.
Note:	They must combine $2(4)$ and their $+c$ correctly to give $+k$
Note:	Going from $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c$ to $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$, with no intermediate working or with no incorrect working is required for the final A1* mark.
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + c + 8$ and stating $k = c+8$
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + k = -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$
	Alternative (integration by parts) method for the 2nd M, 3rd M and 1st A mark
	$\left\{\int \frac{-2(4-u)}{u}\mathrm{d}u = \int \frac{2u-8}{u}\mathrm{d}u\right\} = (2u-8)\ln u - \int 2\ln u\,\mathrm{d}u = (2u-8)\ln u - 2(u\ln u - u)\ \{+c\}$
2nd M1:	Proceeds to obtain an integral of the form $(Au+B)\ln u - \int A\ln u\{\mathrm{d}u\}$; $A, B \neq 0$
3rd M1:	Integrates to give $D\ln u + Eu$; $D, E \neq 0$; which can be simplified or un-simplified with or without a constant of integration.
Note:	Give 3rd M1 for $(2u-8)\ln u - 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$
1st A1:	Integrates to give $(2u-8)\ln u - 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e. with or without a constant of integration.
(b)	
M1: **Note:**	Uses the context of the model and has an understanding that the tree keeps growing until $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 4-\sqrt{h} = 0$. Alternatively, they can write $\frac{\mathrm{d}h}{\mathrm{d}t} > 0 \Rightarrow 4-\sqrt{h} > 0$ Accept $h = 16$ or 16 used in their inequality statement for this mark.
A1:	See scheme
Note:	A correct answer can be given M1 A1 from any working.

Notes for Question 14	
(c)	**Way 1**
B1:	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Condone absence of integral signs.
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$
A1:	Correct integration. E.g. $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}) = \frac{4}{5}t^{1.25}$ $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$ with or without a constant of integration, e.g. k, c or A
Note:	There is no requirement for modulus signs.
M1:	Some evidence of ***applying*** both $t=0$ and $h=1$ to their model (which can be a changed equation) which contains a constant of integration, e.g. k, c or A
dM1:	**dependent on the previous M mark** Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation
M1:	Rearranges their equation to make $t^{\text{their } 1.25} = ...$ followed by a correct method to give $t = ...$; $t > 0$
Note:	$t^{\text{their } 1.25} = ...$ can be negative, but their '$t = ...$' must be positive
Note:	"their 1.25" cannot be 0 or 1 for this mark
Note:	Do not give this mark if $t^{\text{their } 1.25} = ...$ (usually $t^{0.25} = ...$) is a result of substituting $t=12$ (or $t=11$) into the given $\frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. **Note:** They will usually write $\frac{dh}{dt}$ as either 12 or 11.
A1:	awrt 75.2
(c)	**Way 2**
B1:	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working.
Note:	Integral signs and limits are not required for this mark.
M1:	Same as Way 1 (ignore limits)
A1:	Same as Way 1 (ignore limits)
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in h) and subtracts
dM1	**dependent on the previous M mark** Complete process of applying limits of 1 and 12 and 0 and T (or 't') appropriately to their changed equation
M1:	Same as Way 1
A1:	Same as Way 1