Question 14 (Total 15 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$dh = -2(6 - u)\,du$	B1	This mark is given for finding an expression for dh
	$\displaystyle\int \frac{dh}{6 - \sqrt{h}} = \int \frac{-2(6 - u)\,du}{6 - \sqrt{h}}$	M1	This mark is given for substituting $u = 6 - \sqrt{h}$ into the integral
	$\displaystyle = \int -\frac{12}{u} + 2 \;du$	M1	This mark is given for a method to find a simplified version of the integral
	$-12 \ln u + 2u + c$ $= -12 \ln (6 - \sqrt{h}) + 2(6 - \sqrt{h}) + c$	M1	This mark is given for integrating with respect to u to produce an expression in terms of h
		A1	This mark is given for a correct expression for the integral
	$= -12 \ln (6 - \sqrt{h}) - 2\sqrt{h} + k$	A1	This mark is given for a full proof to arrive at the answer as shown (appreciating that $k = c + 12$)
(b)	$\dfrac{dh}{dt} = 0 \;\Rightarrow\; 6 - \sqrt{h} = 0$	M1	This mark is given for a setting $\dfrac{dh}{dt} = 0$
	$0 < h < 36$	A1	This mark is given for deducing the range of the heights of the trees for this model
(c)	$\dfrac{dh}{dt} = \dfrac{t^{0\,25}(4 - \sqrt{h})}{20} \;\Rightarrow\; \dfrac{dh}{(4 - \sqrt{h})} = \dfrac{t^{0\,25}dt}{20}$	B1	This mark is given for separating the variables
	$-12 \ln (6 - \sqrt{h}) - 2\sqrt{h} + k = \dfrac{t^{1\,25}}{25}$	M1	This mark is given for a method to integrate both sides of the equation
		A1	This mark is given for integrating both sides of the equation correctly
	When $t = 0$ and $h = 1$, $-12 \ln 5 - 2 + k = 0$ $k = 2 + 12 \ln 5$	M1	This mark is given for substituting values of $t = 0$ and $h = 1$ to find a value for k
	When $h = 15$, $-12 \ln (6 - \sqrt{15}) - 2\sqrt{15} + 2 + 12 \ln 5 = \dfrac{t^{1\,25}}{25}$	M1	This mark is given for a method to find a value for t by substituting $h = 15$ into the equation

Part	Working or answer an examiner might expect to see	Mark	Notes
	$t^{1\,25} = 112.7661\ldots \;\Rightarrow\; t = \sqrt[1\,25]{112.7661}$	M1	This mark is given for simplifying to find an expression for t
	$t = 43.83$ years	A1	This mark is given for correctly finding the time the tree would take to reach a height of 15 metres