11.

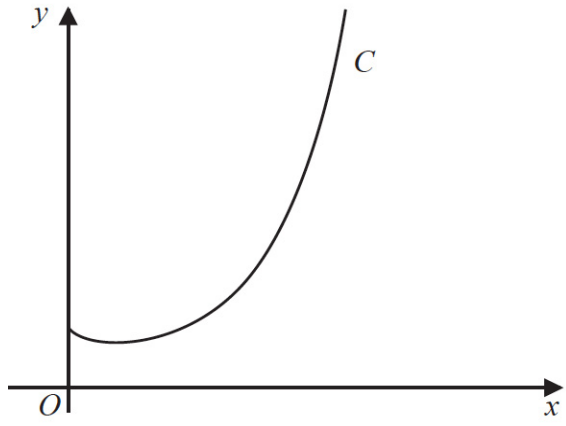

Figure 8

Figure 8 shows a sketch of the curve C with equation

$$y = x^x, \quad x > 0.$$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$.

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}.$$

Using this formula with $x_1 = 1.5$,

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n.

(2)

(Total for Question 11 is 11 marks)