Question	Scheme	Marks	AOs
10 (a)	$\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$	M1	3.1a
	$=\left(2\cos^2 A-1\right)\cos A-\left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$=\left(2\cos^2 A-1\right)\cos A-2\cos A\left(1-\cos^2 A\right)$	ddM1	2.1

	$= 4\cos^3 A - 3\cos A*$	A1*	1.1b
		(4)	
(b)	$1-\cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ $\Rightarrow \cos x = \ldots$	dM1	3.1a
	Two of $-90°, 0, 90°$, awrt $139°$	A1	1.1b
	All four of $-90°, 0, 90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

Allow a proof in terms of *x* rather than A

M1: Attempts to use the compound angle formula for cos(2*A* + *A*) or cos(*A* + 2*A*)
Condone a slip in sign

dM1: Uses correct double angle identities for cos 2*A* and sin 2*A*
$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.
Depends on previous mark.

ddM1: Attempts to get all terms in terms of cos *A* using correct and appropriate identities.
Depends on both previous marks.

A1*: A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.

Alternative right to left is possible:

$$4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right) = \cos A\left(2\cos^2 A - 1 + 2\left(1-\sin^2 A\right) - 2\right) = \cos A\left(\cos 2A - 2\sin^2 A\right)$$

$$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$$

Score M1: For $4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right)$

dM1: For $\cos A\left(2\cos^2 A - 1 + 2\left(1-\sin^2 A\right) - 2\right)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2\left(1-\sin^2 A\right)$)

ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$

A1: $\cos(2A + A) = \cos 3A$

(b)

M1: For an attempt to produce an equation just in cos *x* using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$
Allow one slip in sign or coefficient when copying the result from part (a)

dM1: Dependent upon the preceding mark. It is for taking the cubic equation in cos *x* and making a valid attempt to solve. This could include factorisation or division of a cos *x* term followed by an attempt to solve the 3 term quadratic equation in cos *x* to reach at least one non zero value for cos *x*.
May also be scored for solving the cubic equation in cos *x* to reach at least one non zero value for cos *x*.

A1: Two of $-90°, 0, 90°$, awrt $139°$ **Depends on the first method mark**.

A1: All four of $-90°, 0, 90°$, awrt $139°$ with no extra solutions offered within the range.

Note that this is an alternative approach for obtaining the cubic equation in (b):

$$1-\cos 3x=\sin^2 x \Rightarrow 1-\cos 3x=\frac{1}{2}(1-\cos 2x)$$

$$\Rightarrow 2-2\cos 3x=1-\cos 2x$$

$$\Rightarrow 1=2\cos 3x-\cos 2x$$

$$\Rightarrow 1=2\left(4\cos^3 x-3\cos x\right)-\left(2\cos^2 x-1\right)$$

$$\Rightarrow 0=4\cos^3 x-3\cos x-\cos^2 x$$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for cos $2x$