

Question	Scheme	Marks	AOs
12(a)(i)	$y \times \frac{dx}{dt} = 5 \sin 2t \times 6 \cos t \quad \text{or} \quad 5 \times 2 \sin t \cos t \times 6 \cos t$	M1	1.2

	$\text{(Area =)} \int 5 \sin 2t \times 6 \cos t \, dt = \int 5 \times 2 \sin t \cos t \times 6 \cos t \, dt$ <p style="text-align: center;">or</p> $\int 5 \sin 2t \times 6 \cos t \, dt = \int 60 \sin t \cos^2 t \, dt$	dM1	1.1b
	$\text{(Area =)} \int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt \quad *$	A1*	2.1*
		(3)	
(a)(ii)	$\int 60 \sin t \cos^2 t \, dt = -20 \cos^3 t$	M1 A1	1.1b 1.1b
	$\text{Area} = \left[-20 \cos^3 t \right]_0^{\frac{\pi}{2}} = 0 - (-20) = 20 \quad *$	A1*	2.1
		(3)	
(b)	$5 \sin 2t = 4.2 \Rightarrow \sin 2t = \frac{4.2}{5}$	M1	3.4
	$t = 0.4986\dots, 1.072\dots$	A1	1.1b
	Attempts to find the x values at both t values	dM1	3.4
	$t = 0.4986\dots \Rightarrow x = 2.869\dots$ $t = 1.072 \Rightarrow x = 5.269\dots$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
(11 marks)			

Notes:

(a)(i)

M1: Attempts to multiply y by $\frac{dx}{dt}$ to obtain $A \sin 2t \cos t$ but may apply $\sin 2t = 2 \sin t \cos t$ here

dM1: Attempts to use $\sin 2t = 2 \sin t \cos t$ within an integral which may be implied by

$$\text{e.g. } A \int \sin 2t \times \cos t \, dt = \int k \sin t \cos^2 t \, dt$$

A1*: Fully correct work leading to the given answer.

This must include $\sin 2t = 2 \sin t \cos t$ or e.g. $5 \sin 2t = 10 \sin t \cos t$ seen explicitly in their proof and a correct intermediate line that includes an integral sign and the “ dt ”

Allow the limits to just “appear” in the final answer e.g. working need not be shown for the limits.

(a)(ii)

M1: Obtains $\int 60 \sin t \cos^2 t \, dt = k \cos^3 t$. This may be attempted via a substitution of $u = \cos t$ to obtain

$$\int 60 \sin t \cos^2 t \, dt = k u^3$$

A1: Correct integration $-20 \cos^3 t$ or equivalent e.g. $-20u^3$

A1*: Rigorous proof with all aspects correct including the correct limits and the $0 - (-20)$ and

not just: $-20 \cos^3 \frac{\pi}{2} - (-20 \cos^3 0) = 20$

(b)

M1: Uses the given model and attempts to find value(s) of t when $\sin 2t = \frac{4.2}{5}$. Look for $2t = \sin^{-1} \frac{4.2}{5} \Rightarrow t = \dots$

A1: At least one correct value for t , correct to 2 dp. FYI $t = 0.4986\dots, 1.072\dots$ or in degrees $t = 28.57\dots, 61.42\dots$

dM1: Attempts to find **TWO** distinct values of x when $\sin 2t = \frac{4.2}{5}$. Condone poor trig work and allow this mark if 2

values of x are attempted from 2 values of t .

A1: Both values correct to 2 dp. NB $x = 2.869\dots, 5.269\dots$

Or may take Cartesian approach

$$5 \sin 2t = 4.2 \Rightarrow 10 \sin t \cos t = 4.2 \Rightarrow 10 \frac{x}{6} \sqrt{1 - \frac{x^2}{36}} = 4.2 \Rightarrow x^4 - 36x^2 + 228.6144 = 0 \Rightarrow x = 2.869\dots, 5.269\dots$$

M1: For converting to Cartesian form A1: Correct quartic M1: Solves quartic A1: Correct values

A1: 2.40 metres or 240 cm

Allow awrt 2.40 m or allow 2.4m (not awrt 2.4 m) and allow awrt 240 cm. **Units are required.**