

Question	Scheme	Marks	AOs
13(a)	$k = e^2$ or $x \neq e^2$	B1	2.2a
		(1)	

(b)	$g'(x) = \frac{(\ln x - 2) \times \frac{3}{x} - (3 \ln x - 7) \times \frac{1}{x}}{(\ln x - 2)^2} = \frac{1}{x(\ln x - 2)^2}$ <p style="text-align: center;">or</p> $g'(x) = \frac{d}{dx} \left(3 - (\ln(x) - 2)^{-1} \right) = (\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$ <p style="text-align: center;">or</p> $g'(x) = (\ln x - 2)^{-1} \times \frac{3}{x} - (3 \ln x - 7)(\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) AND $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3 \ln x - 7 \dots 0$ or $\ln x - 2 \dots 0$ or $3 \ln a - 7 \dots 0$ or $\ln a - 2 \dots 0$ where \dots is “=” or “>” to reach a value for x or a but may be seen as an inequality e.g. $x > \dots$ or $a > \dots$	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	
(6 marks)			

Notes:

(a)

B1: Deduces $k = e^2$ or $x \neq e^2$ Condone $k = \text{awrt } 7.39$ or $x \neq \text{awrt } 7.39$

(b)

M1: Attempts to differentiate via the quotient rule and with $\ln x \rightarrow \frac{1}{x}$ so allow for:

$$\frac{d}{dx}(g(x)) = \frac{(\ln x - 2) \times \frac{\alpha}{x} - (3 \ln x - 7) \times \frac{\beta}{x}}{(\ln x - 2)^2}, \quad \beta > 0$$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively attempts to write $g(x) = \frac{3 \ln(x) - 7}{\ln(x) - 2} = 3 - (\ln(x) - 2)^{-1}$ and attempts the chain rule so allow for:

$$3 - (\ln(x) - 2)^{-1} \rightarrow (\ln(x) - 2)^{-2} \times \frac{\alpha}{x}$$

Alternatively writes $g(x) = (3 \ln(x) - 7)(\ln(x) - 2)^{-1}$ and attempts the product rule so allow for:

$$g'(x) = (\ln x - 2)^{-1} \times \frac{\alpha}{x} - (3 \ln x - 7)(\ln x - 2)^{-2} \times \frac{\beta}{x}$$

In general condone missing brackets for the M mark. E.g. if they quote $u = 3 \ln x - 7$ and $v = \ln x - 2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: $\frac{1}{x(\ln x - 2)^2}$ Allow $\frac{\frac{1}{x}}{(\ln x - 2)^2}$ i.e. we need to see the numerator simplified to $1/x$

Note that some candidates establish the correct numerator and correct denominator independently and provided they obtain the correct expressions, this mark can be awarded.

But allow a correctly expanded denominator.

A1cso: States that as $x > 0$ **AND** $\ln x - 2$ is squared so $g'(x) > 0$

(c)

M1: Attempts to solve either $3\ln x - 7 = 0$ or $\ln x - 2 = 0$ or using inequalities e.g. $3\ln x - 7 > 0$

A1: $0 < a < e^2, a > e^{\frac{7}{3}}$