Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k + 1$, $3k + 2$ or equivalent e.g. $3k - 1$, $3k$, $3k + 1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3 \times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b

	$(3k+1)^2 = 9k^2+6k+1 = 3\times(3k^2+2k)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times(3k^2+4k+1)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times(3k^2-2k)+1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k+1$, $3k+2$ or e.g. $3k-1$, $3k$, $3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 <u>using algebra</u>. This must be made explicit e.g. reaches $3\times(3k^2+2k)+1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2 + 6k + 1$ is one more than a multiple of 3 e.g. "$9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2 + 6k + 1$ is one more than a multiple of 3"

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion