

Question	Scheme	Marks	AOs
12(a)	$u = 1 + \sqrt{x} \Rightarrow x = (u-1)^2 \Rightarrow \frac{dx}{du} = 2(u-1)$ <p style="text-align: center;">or</p> $u = 1 + \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\int \frac{x}{1+\sqrt{x}} dx = \int \frac{(u-1)^2}{u} 2(u-1) du$ <p style="text-align: center;">or</p> $\int \frac{x}{1+\sqrt{x}} dx = \int \frac{x}{u} \times 2x^{\frac{1}{2}} du = \int \frac{2x^{\frac{3}{2}}}{u} du = \int \frac{2(u-1)^3}{u} du$	M1	2.1
	$\int_0^{16} \frac{x}{1+\sqrt{x}} dx = \int_1^5 \frac{2(u-1)^3}{u} du$	A1	1.1b
	(3)		
(b)	$2 \int \frac{u^3 - 3u^2 + 3u - 1}{u} du = 2 \int \left(u^2 - 3u + 3 - \frac{1}{u} \right) du = \dots$	M1	3.1a
	$= (2) \left[\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u \right]$	A1	1.1b
	$= 2 \left[\frac{5^3}{3} - \frac{3(5)^2}{2} + 3(5) - \ln 5 - \left(\frac{1}{3} - \frac{3}{2} + 3 - \ln 1 \right) \right]$	dM1	2.1
	$= \frac{104}{3} - 2 \ln 5$	A1	1.1b
	(4)		

(7 marks)

Notes

(a)

B1: Correct expression for $\frac{dx}{du}$ or $\frac{du}{dx}$ (or u') or dx in terms of du or du in terms of dx

M1: Complete method using the given substitution.

This needs to be a correct method for their $\frac{dx}{du}$ or $\frac{du}{dx}$ leading to an integral in terms of u

only (ignore any limits if present) so for each case you need to see:

$$\frac{dx}{du} = f(u) \rightarrow \int \frac{x}{1+\sqrt{x}} dx = \int \frac{(u-1)^2}{u} f(u) du$$

$$\frac{du}{dx} = g(x) \rightarrow \int \frac{x}{1+\sqrt{x}} dx = \int \frac{x}{u} \times \frac{du}{g(x)} = \int h(u) du. \text{ In this case you can condone}$$

$$\text{slips with coefficients e.g. allow } \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow \int \frac{x}{1+\sqrt{x}} dx = \int \frac{x}{u} \times \frac{x^{\frac{1}{2}}}{2} du = \int h(u) du$$

$$\text{but not } \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow \int \frac{x}{1+\sqrt{x}} dx = \int \frac{x}{u} \times \frac{x^{-\frac{1}{2}}}{2} du = \int h(u) du$$

A1: All correct with correct limits and no errors. The “du” must be present but may have been omitted along the way but it must have been seen at least once before the final answer. The limits can be seen as part of the integral or stated separately.

(b)

M1: Realises the requirement to cube the bracket and divide through by u and makes progress with the integration to obtain at least 3 terms of the required form e.g. 3 from $ku^3, ku^2, ku, k \ln u$

A1: Correct integration. This mark can be scored with the “2” still outside the integral or even if it has been omitted. But if the “2” has been combined with the integrand, the integration must be correct.

dM1: Completes the process by applying their “changed” limits and subtracts the right way round

Depends on the first method mark.

A1: Cao (Allow equivalents for $\frac{104}{3}$ e.g. $\frac{208}{6}$)