

4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

FINEVIEW

where a , b and c are integers to be found.

(3)

Small angle approximations:
in radians,

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$\text{so, } 4 \sin \frac{\theta}{2} \approx 4 \left(\frac{\theta}{2} \right) = 2\theta$$

$$\begin{aligned} 3 \cos^2 \theta &\approx 3 \left(1 - \frac{\theta^2}{2} \right)^2 = 3 \left(1 - \theta^2 + \frac{\theta^4}{4} \right) \quad (1 \text{ mark}) \\ &= 3 - 3\theta^2 + \frac{3}{4}\theta^4 \end{aligned}$$

$$\text{so, } 4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx 2\theta + 3 - 3\theta^2 + \frac{3}{4}\theta^4 \quad (1 \text{ mark})$$

because θ is small, θ^4 is very small and $\rightarrow 0$, so

$$\approx 3 + 2\theta - 3\theta^2 \quad (1 \text{ mark})$$