



5. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \quad x \in \mathbb{R}$$

(a) Find

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

(a)(i) $\frac{dy}{dx} = 4(5)x^{4-1} - 3(24)x^{3-1} + 2(42)x^{2-1} - 32$ (1mark)

$$= 20x^3 - 72x^2 + 84x - 32$$
 (1mark)

(a)(ii) $\frac{d^2y}{dx^2} = 3(20)x^{3-1} - 2(72)x^{2-1} + 84$

$$= 60x^2 - 144x + 84$$
 (1mark)

(b)(i) when $x=1$, $\frac{dy}{dx} = 20(1)^3 - 72(1)^2 + 84(1) - 32$ (1mark)

$$= 0 \quad \text{so } C \text{ has stationary point at } x=1 \text{ (1mark)}$$

(b)(ii) when $x=1$, $\frac{d^2y}{dx^2} = 60(1)^2 - 144(1) + 84$

$= 0$ so this could be an inflection point
we need to test points on either side

when $x=0.9$, $\frac{d^2y}{dx^2} = 60(0.9)^2 - 144(0.9) + 84 = 3 > 0$

when $x=1.1$, $\frac{d^2y}{dx^2} = 60(1.1)^2 - 144(1.1) + 84 = -1.8 < 0$

$\frac{d^2y}{dx^2}$ changes sign (from >0 convex to <0 concave)

so stationary point at $x=1$ is point of inflection (2marks)