



Figure 1

The shape $OABCDEFO$ shown in Figure 1 is a design for a logo.

In the design

- OAB is a sector of a circle centre O and radius r
- sector OFE is congruent to sector OAB
- ODC is a sector of a circle centre O and radius $2r$
- AOF is a straight line

Given that the size of angle COD is θ radians,

(a) write down, in terms of θ , the size of angle AOB

$$\text{(a) } \angle AOB = \frac{\pi - \theta}{2} \quad (1 \text{ mark}) \quad (1)$$

(b) Show that the area of the logo is

$$\frac{1}{2} r^2 (3\theta + \pi) \quad (2)$$

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r , θ and π .

(2)

$$\text{(b) Area of Logo} = \text{Area big sector} + 2 (\text{Area small sector})$$

$$\begin{aligned} \left(\begin{array}{l} \text{Area Sector} \\ = \frac{1}{2} r^2 \theta \end{array} \right) &= \frac{1}{2} (2r)^2 \theta + 2 \left(\frac{1}{2} r^2 \left(\frac{\pi - \theta}{2} \right) \right) \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} &= 2r^2 \theta + \frac{r^2 (\pi - \theta)}{2} = \frac{1}{2} (4r^2 \theta + r^2 \pi - r^2 \theta) \\ &= \frac{1}{2} r^2 (3\theta + \pi) \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{(c) Perimeter of Logo} &= 4r + \text{Length large arc} + 2 (\text{Length small arc}) \\ \left(\begin{array}{l} \text{Length Arc} \\ = r\theta \end{array} \right) &= 4r + 2r\theta + 2 \left(r \left(\frac{\pi - \theta}{2} \right) \right) \quad (1 \text{ mark}) \\ &= 4r + 2r\theta + r(\pi - \theta) = 4r + 2r\theta + r\pi - r\theta \\ &= r(4 + \pi + \theta) \quad (1 \text{ mark}) \end{aligned}$$