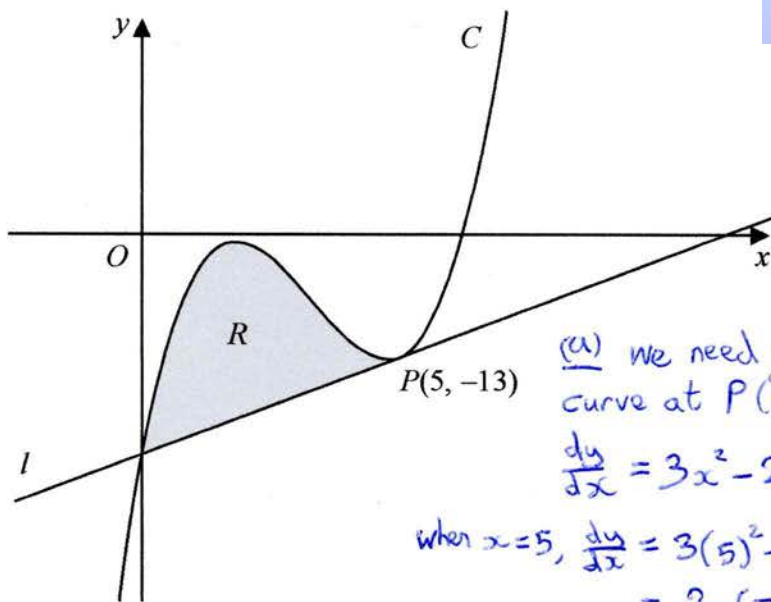


In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.



(a) we need gradient of curve at $P(5, -13)$
 $\frac{dy}{dx} = 3x^2 - 20x + 27$ (1 mark)
 when $x=5$, $\frac{dy}{dx} = 3(5)^2 - 20(5) + 27 = 2 (=m)$ (1 mark)

Figure 2

Equation of line is

$$\frac{y - y_1}{x - x_1} = m \Rightarrow \frac{y - (-13)}{x - 5} = 2$$

$$y = x^3 - 10x^2 + 27x - 23$$

$$\Rightarrow y = 2x - 23 \text{ (2 marks)}$$

Figure 2 shows a sketch of part of the curve C with equation

The point $P(5, -13)$ lies on C

(b) when $x=0$, Curve $y = 0^3 - 10(0)^2 + 27(0) - 23 = -23$
 when $x=0$, line $y = 2(0) - 23 = -23$
 so Curve & line also meet on y -axis (at $(0, -23)$) (1 mark)

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l , giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y -axis.

(1)

The finite region R , shown shaded in Figure 2, is bounded by the curve C and the line l .

(c) Use algebraic integration to find the exact area of R .

Curve is above Line for R

(4)

(c) Easiest way to find R is to integrate (Curve - Line) between points of intersection:

$$\int_0^5 (x^3 - 10x^2 + 27x - 23) - (2x - 23) dx$$

$$= \int_0^5 x^3 - 10x^2 + 25x dx = \left[\frac{x^4}{4} - \frac{10x^3}{3} + \frac{25x^2}{2} \right]_0^5 \text{ (2 marks)}$$

$$= \left(\frac{5^4}{4} - \frac{10(5)^3}{3} + \frac{25(5)^2}{2} \right) - (0 - 0 + 0) = \frac{625}{12} \text{ (2 marks)}$$