

8. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

(a) x & y are mixed up, so we need to do
Implicit Differentiation

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

$$px^3 + qxy + 3y^2 = 26$$

$$3px^2 + q(y + x\frac{dy}{dx}) + 6y\frac{dy}{dx} = 0$$

(Product Rule) (Chain Rule)
(2 marks)

where a , b and c are integers to be found.

Given that

• the point $P(-1, -4)$ lies on C

• the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q .

(a) contd Now, we need to make $\frac{dy}{dx}$ the subject (4)

$$3px^2 + qy + qx\frac{dy}{dx} + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(qx + 6y) = -3px^2 - qy \quad (1 \text{ mark})$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3px^2 - qy}{qx + 6y} \quad (1 \text{ mark})$$

(5)

(b) Given $P(-1, -4)$ lies on C ,

$$p(-1)^3 + q(-1)(-4) + 3(-4)^2 = 26$$

$$-p + 4q + 48 = 26$$

$$-p + 4q = -22 \quad (1 \text{ mark})$$

Given Normal to C at P is $19x + 26y + 123 = 0$

$$\Rightarrow 26y = -19x - 123$$

$$y = -\frac{19}{26}x - \frac{123}{26}$$

gradient of Normal at P \nearrow (1 mark)

$$\text{Gradient of Tangent at } P = -\left(\frac{-19}{26}\right) = \frac{19}{26}$$

$$\text{So, } \frac{dy}{dx} = \frac{19}{26} = \frac{-3p(-1)^2 - q(-4)}{q(-1) + 6(-4)} = \frac{-3p + 4q}{-q - 24} \quad (1 \text{ mark})$$

$$\Rightarrow 26(-q - 24) = 19(-3p + 4q) \Rightarrow -26q - 624 = -57p + 76q$$

$$\Rightarrow 57p - 102q = 624 \Rightarrow 19p - 34q = 208$$

$$\left. \begin{array}{l} -p + 4q = -22 \\ 19p - 34q = 208 \end{array} \right\} \text{ Question says "find" values, so could use Calculator to solve Simultaneous Equations } \Rightarrow p = 2, q = -5 \quad (2 \text{ marks})$$