

9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

Let's consider a few terms:

$$n=2 \Rightarrow \left(\frac{3}{4}\right)^2 \cos(180(2)) = \left(\frac{3}{4}\right)^2 (1) = \left(\frac{3}{4}\right)^2$$

$$n=3 \Rightarrow \left(\frac{3}{4}\right)^3 \cos(180(3)) = \left(\frac{3}{4}\right)^3 (-1) = -\left(\frac{3}{4}\right)^3$$

$$n=4 \Rightarrow \left(\frac{3}{4}\right)^4 \cos(180(4)) = \left(\frac{3}{4}\right)^4 (1) = \left(\frac{3}{4}\right)^4$$

so we have a geometric sequence with

$$a = \left(\frac{3}{4}\right)^2 = \frac{9}{16} \quad r = -\frac{3}{4} \quad (1 \text{ mark})$$

$$|r| < 1 \text{ so } S_{\infty} = \frac{a}{1-r}$$

$$= \frac{\frac{9}{16}}{1 - -\frac{3}{4}} \quad (1 \text{ mark})$$

$$= \frac{\frac{9}{16}}{\frac{7}{4}} = \frac{9}{16} \times \frac{4}{7}$$

$$= \frac{9}{28} \quad (1 \text{ mark})$$