



10. The time,  $T$  seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where  $l$  metres is the length of the pendulum and  $a$  and  $b$  are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b \log_{10} l + \log_{10} a$$

(2)

(a)  $T = al^b$

$\log_{10} T = \log_{10}(al^b)$   
 $\log_{10} T = \log_{10} a + \log_{10} l^b$  (1 mark)  
 $\log_{10} T = \log_{10} a + b \log_{10} l$  (1 mark)  
 $= b \log_{10} l + \log_{10} a$

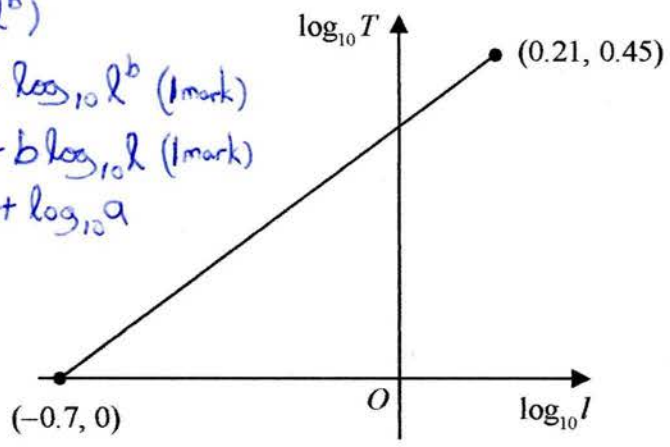


Figure 3

A student carried out an experiment to find the values of the constants  $a$  and  $b$ .

The student recorded the value of  $T$  for different values of  $l$ .

Figure 3 shows the linear relationship between  $\log_{10} l$  and  $\log_{10} T$  for the student's data.

The straight line passes through the points  $(-0.7, 0)$  and  $(0.21, 0.45)$

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

(c) when  $l=1$ ,  $T = a(1)^b = a$   
 so  $a$  is the time it takes  
 for one swing of a pendulum  
 of length 1 m. (1 mark)

giving the value of  $a$  and the value of  $b$ , each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant  $a$ .

(1)

(b) when  $\log_{10} T = 0$ ,  $\log_{10} l = -0.7$ , so  $0 = -0.7b + \log_{10} a$   
 when  $\log_{10} T = 0.45$ ,  $\log_{10} l = 0.21$ , so  $0.45 = 0.21b + \log_{10} a$   
 $\log_{10} a = 0.7b = 0.45 - 0.21b \Rightarrow 0.91b = 0.45 \Rightarrow b = \frac{45}{91}$  (1 mark)  
 $\log_{10} a = 0.7 \left(\frac{45}{91}\right) \Rightarrow a = 10^{0.7 \left(\frac{45}{91}\right)} = 2.218... = 2.22$  3sf (1 mark)  
 so  $T = 2.22 l^{0.495}$   $\left(\frac{45}{91}\right)$  3sf (1 mark)