

12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}} dx = \int_p^q \frac{2(u-1)^3}{u} du$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}} dx = A - B \ln 5$$

where A and B are constants to be found.

(4)

(a) we need to replace dx with du :

$$u = 1 + x^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} \quad (1 \text{ mark})$$

$$\Rightarrow dx = 2x^{\frac{1}{2}} du \quad \text{so, } \int_{x=0}^{x=16} \frac{x \times 2x^{\frac{1}{2}}}{u} du$$

$$= \int_{x=0}^{x=16} \frac{2x^{\frac{3}{2}}}{u} du \quad u = 1 + x^{\frac{1}{2}} \Rightarrow x^{\frac{1}{2}} = u - 1 \Rightarrow x^{\frac{3}{2}} = (u-1)^3$$

$$\text{so, } = \int_{x=0}^{x=16} \frac{2(u-1)^3}{u} du \quad (1 \text{ mark})$$

Now, change limits

x	$u = 1 + \sqrt{x}$
0	1
16	5

$$\text{so, } = \int_{u=1}^{u=5} \frac{2(u-1)^3}{u} du \quad (1 \text{ mark})$$

$$(b) = \int_1^5 \frac{2u^3 - 6u^2 + 6u - 2}{u} du$$

$$= \int_1^5 (2u^2 - 6u + 6 - \frac{2}{u}) du \quad (1 \text{ mark})$$

$$\left(\begin{aligned} (u-1)(u-1)^2 &= (u-1)(u^2 - 2u + 1) \\ &= u^3 - 2u^2 + u - u^2 + 2u - 1 \\ &= u^3 - 3u^2 + 3u - 1 \end{aligned} \right)$$

$$= \left[\frac{2}{3}u^3 - 3u^2 + 6u - 2\ln u \right]_1^5 \quad (1 \text{ mark})$$

$$= \left(\frac{2}{3}(125) - 3(25) + 6(5) - 2\ln 5 \right) - \left(\frac{2}{3}(1)^3 - 3(1)^2 + 6(1) - 2\ln(1) \right) \quad (1 \text{ mark})$$

$$= \left(\frac{250}{3} - 75 + 30 - 2\ln 5 \right) - \left(\frac{2}{3} - 3 + 6 - 2(0) \right)$$

$$= \frac{104}{3} - 2\ln 5 \quad (1 \text{ mark})$$