

13. The curve  $C$  has parametric equations

$$x = \sin 2\theta \quad y = \operatorname{cosec}^3 \theta \quad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for  $\frac{dy}{dx}$  in terms of  $\theta$

(3)

(b) Hence find the exact value of the gradient of the tangent to  $C$  at the point where  $y = 8$

(3)

(a)  $\frac{dy}{d\theta} = 3 \operatorname{cosec}^2 \theta \times -\operatorname{cosec} \theta \cot \theta$  by Chain Rule (1 mark)

$$= -3 \operatorname{cosec}^3 \theta \cot \theta \quad \left( \begin{array}{l} \text{derivative of } \operatorname{cosec} \theta \\ \text{given in Formula Book} \end{array} \right)$$

$$\frac{dx}{d\theta} = \cos 2\theta \times 2 \text{ by Chain Rule} = 2 \cos 2\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \quad (1 \text{ mark})$$

$$= \frac{-3 \operatorname{cosec}^3 \theta \cot \theta}{2 \cos 2\theta} \quad (1 \text{ mark})$$

(b) when  $y = 8$ ,  $\operatorname{cosec}^3 \theta = 8$

$$\operatorname{cosec} \theta = (8)^{\frac{1}{3}} = 2$$

$$\frac{1}{\sin \theta} = 2 \Rightarrow \sin \theta = \frac{1}{2} \quad (1 \text{ mark})$$

In range  $0 < \theta < \frac{\pi}{2}$ ,  $\sin^{-1}(\frac{1}{2}) = \theta = \frac{\pi}{6}$

so,  $\frac{dy}{dx} = \frac{-3 \operatorname{cosec}^3(\frac{\pi}{6}) \cot(\frac{\pi}{6})}{2 \cos(\frac{2\pi}{6})} \quad (1 \text{ mark})$

$$= \frac{-\frac{3}{\sin^3(\frac{\pi}{6})} \left( \frac{\cos(\frac{\pi}{6})}{\sin(\frac{\pi}{6})} \right)}{2 \cos(\frac{\pi}{3})} = \frac{-\frac{3}{\frac{1}{8}} \left( \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \right)}{2 \left( \frac{1}{2} \right)}$$

$$= \frac{-24\sqrt{3}}{1} = -24\sqrt{3} \quad (1 \text{ mark})$$