



Figure 5

(a) $\frac{dV}{dt} = \text{inflow} - \text{outflow}$
 $= 0.48 - 0.1h$ (1 mark)

required equation has no V . We need to eliminate V .

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

$V = 3 \times 8 \times h = 24h$

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

$\Rightarrow \frac{dV}{dh} = 24$ (1 mark)

At time t minutes after the tap has been opened

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{24}(0.48 - 0.1h)$ (1 mark)

- the depth of water in the tank is h metres

$\Rightarrow 1200 \frac{dh}{dt} = 24 - 5h$ (1 mark)

- water is flowing into the tank at a constant rate of 0.48 m^3 per minute

- water is modelled as leaving the tank through the tap at a rate of $0.1h \text{ m}^3$ per minute

(a) Show that, according to the model,

(b) Separating the Variables,
 $\int \frac{1200}{24-5h} dh = \int dt$

$1200 \frac{dh}{dt} = 24 - 5h$
 $\left(\frac{1200}{-5}\right) \ln(24-5h) = t + c$
 $-240 \ln(24-5h) = t + c$ (2 marks) (4)

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

(b) contd Given $h = 2$ when $t = 0$,
 $-240 \ln(24 - 5(2)) = 0 + c \Rightarrow c = -240 \ln(14)$ (1 mark)

$h = A + Be^{-kt}$ so, $-240 \ln(24 - 5h) = t - 240 \ln(14)$ (1 mark)

where A , B and k are constants to be found.

$t = 240 \ln(14) - 240 \ln(24 - 5h)$
 $= 240 \ln\left(\frac{14}{24 - 5h}\right)$ (we want to make h the subject) (6)

Given that the tap remains open,

$\frac{t}{240} = \ln\left(\frac{14}{24-5h}\right) \Rightarrow e^{\frac{t}{240}} = \frac{14}{24-5h} \Rightarrow 24-5h = 14 e^{-\frac{t}{240}}$ (1 mark)

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(b) contd $\Rightarrow h = \frac{24}{5} - \frac{14}{5} e^{-\frac{t}{240}}$ (1 mark) (2)

(c) as $t \rightarrow \infty$, $e^{-\frac{t}{240}} \rightarrow 0$, so $h \rightarrow \frac{24}{5} - \frac{14}{5}(0) = \frac{24}{5} = 4.8 \text{ m}$ (1 mark)

h will never reach 5 m, so tank will never become full (1 mark)