

4. Curve C has equation

$$y = (x+k)(2-x)$$

where k is a constant and $k > 2$

(a) Sketch C, showing the coordinates of any points of intersection with the coordinate axes.

(3)

(b) Find, in simplest form in terms of k , the coordinates of the stationary point of C.

(3)

(a) $y = -x^2 + (2-k)x + 2k$

$$x=0 \Rightarrow y = 2k$$

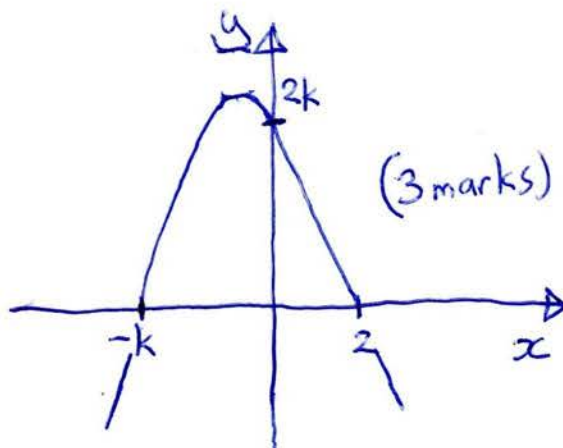
↑
Coeff of x^2
is -ve so

$$y=0 \Rightarrow (x+k)=0 \Rightarrow x=-k$$

$$(2-x)=0 \Rightarrow x=2$$

curve is frowny face '∩'
not smiley face '∪'

($k > 2$ and quadratics are vertically symmetric, so maximum is to left of y-axis)



(b) can find by calculus, by completing the square or by symmetry
by completing the square,

$$-y = x^2 - (2-k)x - 2k$$

$$= x - (2-k)x + \left(\frac{2-k}{2}\right)^2 - \left(\frac{2-k}{2}\right)^2 - 2k$$

$$= \left(x - \frac{2-k}{2}\right)^2 - \left(\frac{2-k}{2}\right)^2 - 2k$$

maximum $-\left(x - \left(\frac{2-k}{2}\right)\right)^2$ is 0

when $x = \frac{2-k}{2}$ (1 mark)

$$y = 0 + \left(\frac{k+2}{2}\right)^2 \quad (1 \text{ mark})$$

so maximum is

$$\left(\frac{2-k}{2}, \left(\frac{k+2}{2}\right)^2\right) \quad (1 \text{ mark})$$

$$+y = -\left(x - \frac{2-k}{2}\right)^2 + \left(\frac{(2-k)^2}{4} + 2k\right)$$

$$= -\left(x - \frac{2-k}{2}\right)^2 + \frac{4-4k+k^2+8k}{4}$$

$$= -\left(x - \frac{2-k}{2}\right)^2 + \frac{k^2+4k+4}{4}$$

$$= -\left(x - \frac{2-k}{2}\right)^2 + \left(\frac{k+2}{2}\right)^2$$