

5. Relative to a fixed origin O ,

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point B has position vector $7\mathbf{i} + \mathbf{j} + 2\mathbf{k}$
- the point C has position vector $4\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$

(a) Find $|\vec{AB}|$ giving your answer as a simplified surd.

(2)

Given that $ABCD$ is a parallelogram,

(b) find the position vector of the point D .

(2)

The point E is positioned such that

- ACE is a straight line
- $AC:CE = 2:1$

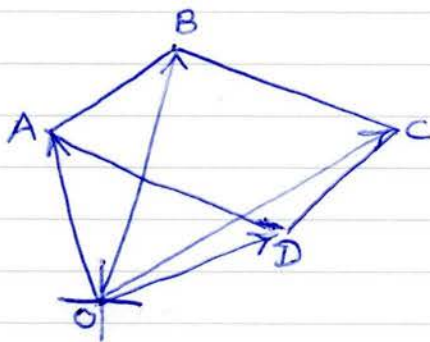
(c) Find the coordinates of the point E .

(2)

(a) $\vec{OA} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$ $\vec{OB} = \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}$ $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} 7-5 \\ 1-3 \\ 2-(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$

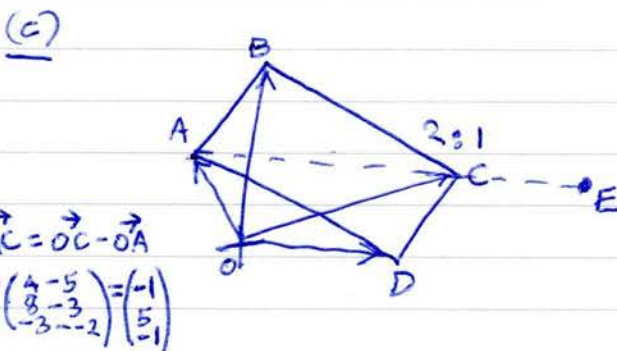
$|\vec{AB}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$ (2 marks)

(b) $\vec{OC} = \begin{pmatrix} 4 \\ 8 \\ -3 \end{pmatrix}$



because $ABCD$ is a parallelogram,
 $\vec{CD} = \vec{BA} = -\vec{AB}$

$\vec{OD} = \vec{OC} + \vec{CD} = \vec{OC} - \vec{AB}$
 $= \begin{pmatrix} 4 \\ 8 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4-2 \\ 8-(-2) \\ -3-4 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \\ -7 \end{pmatrix}$
from (a) (2 marks)



$\vec{AE} = \frac{3}{2}(\vec{AC}) = \frac{3}{2} \begin{pmatrix} -1 \\ 5 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ \frac{15}{2} \\ -\frac{3}{2} \end{pmatrix}$
 $\vec{OE} = \vec{OA} + \vec{AE} = \begin{pmatrix} 5 - \frac{3}{2} \\ 3 + \frac{15}{2} \\ -2 - \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 3\frac{1}{2} \\ 10\frac{1}{2} \\ -3\frac{1}{2} \end{pmatrix}$ (2 marks)